Austrian authors of Tables of Logarithms around 1800

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1 Introduction

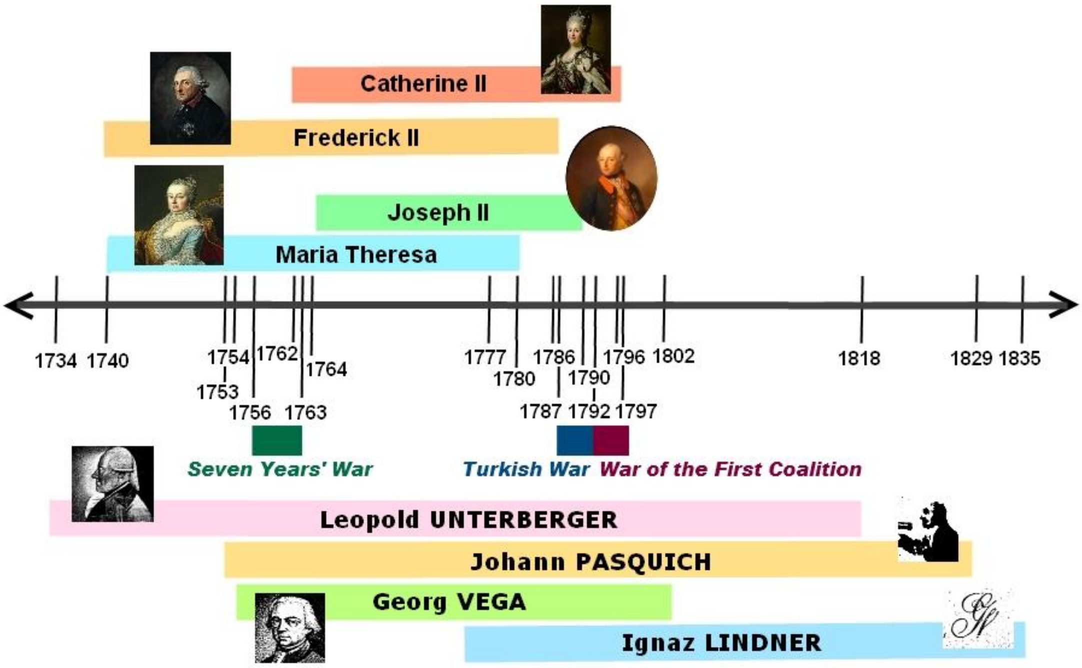


Figure 1: Historical overview

Several tables of logarithms were published by Austrian mathematicians during the politically turbulent time around 1800. The militarist and professor of mathematics Georg of Vega calculated tables of logarithms for pupils, mathematicians, astronomers, navigators, professors and several mathematics fanciers who had to make exact calculations. His contemporary Leopold of Unterberger published tables of logarithms too.
Vega's student Ignaz Lindner supported him with the calculation of logarithms and created his own tables later. The tables of logarithms of Johann Pasquich, a priest and friend of Vega's, are not very well known.

In this paper a short overview of the biographies of Georg of Vega (1754 – 1802), Leopold of Unterberger (1734 – 1818), Ignaz Lindner (1777 – 1835) and Johann Pasquich (1753 – 1829) will be given. Furthermore, their tables of logarithms will be introduced and an insight into the methods of calculation will be given.

2 Georg of Vega

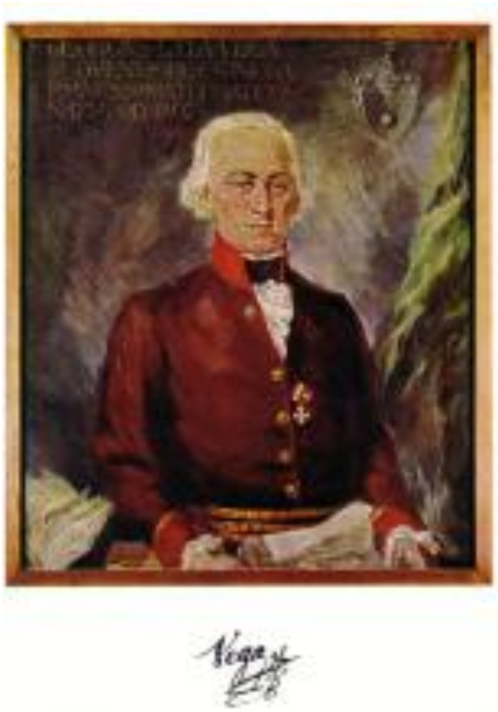


Figure 2: Jurij Vega - Matej Sternen, 1938 University of Pittsburg, ZDA

2.1 Biography

During a politically turbulent time Georg Vega was born in Zagorica (Slovenia) on March 23, 1754.

Figure 3: Vega's birthplace – picture Faustmann, 2014

Queen Maria Theresa was reigning since the year 1740 and the opposition to the Prussian King Frederick II was her most difficult task.
In 1780 Vega entered the artillery and was awarded the lectureship of mathematics in the academy for artillery in Vienna. One year later he got his first military degree.

At that time, the Academy for Artillery did not have any good mathematical textbooks and therefore Vega wrote his first book for his students. Moreover, he thought that advancements in the artillery could only be achieved by deep mathematical knowledge. The first part of his mathematical lectures was published in the year 1782 in Vienna. One year later he published his first tables of logarithms.

Together with his pupils Vega tried to write faultless tables and formulas. Moreover, an aim was to keep the selling-price of his tables considerably low. He recalculated many tables and compared them with the tables of Schulze, Gardiner, Vlacq and Pitisci. In the prologue he promised to pay one ducat for every mistake.

In 1784 Vega was promoted lieutenant and he published the second part of the „Mathematische Vorlesungen“.

The "Bombardierkorps" was founded in the year 1786. It included 4 companies and the commanding officer was Leopold Unterberger. Georg Vega, who received a position as a professor of mathematics, was very much admired by his pupils and well known beyond the army.

Politically, that time was turbulent as well. Tsarina Catherine II intended to dislodge the Turks away from Europe and wanted to win Joseph II as a confederate. At first he hesitated but after Russia had started a war in the year 1787 Austria entered this war (1787 – 1792) one year later.

Georg was expected to stay as a professor in Vienna, but he preferred to serve in the war to test his theoretical knowledge in front of the enemies. During that war he continued his mathematical work and a witness' testimony reported that Georg was calculating logarithms while cannonballs were flying above his head. In 1792 the War of the First Coalition (1792 – 1797) started. One year later Vega was promoted major. In the same year he published the "logarithmisch-trigonometrisches Handbuch" in Leipzig.

He became a member of the Academy of Sciences in Erfurt. During this war Vega sometimes travelled to Vienna and wrote his third book of logarithms. In 1794 he stayed in Stuttgart for two months and probably became acquainted with Schiller's circle. In the same year he became a writing member of the Royal British Society of Sciences in Göttingen and his "Thesaurus logarithmorum completus ex arithmetica logarithmica et ex trigonometria" was published in Leipzig.

This book was written for astronomers, navigators, professors, and all mathematics fanciers who had to make exact calculations.

During the War of the First Coalition Vega noticed that guns would get more power and a longer range if they were adjusted and set up according to the rules of mathematics. For this excellent performance Vega got an attestation from Leopold Unterberger. After mid-September 1802, Vega was missing and on September 26, 1802 his dead body was found in the Danube. There were various opinions about his death which ranged from murder to suicide.

2.2 Vega's tables

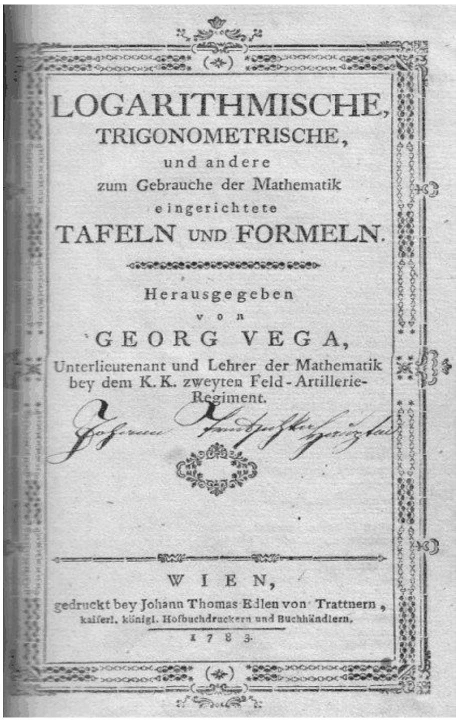

LOGARITHMISCHE,
TRIGONOMETRISCHE,
und andere
zum Gebrauche der Mathematik
eingerichtete
TAFELN UND FORMELN.
Herausgegeben
von
GEORG VEGA,
Unterlieutenant und Lehrer der Mathematik
bey dem K. K. zweyten Feld-Artillerie-
Regiment.
WIEN,
gedruckt bey Johann Thomas Edlen von Trattnern,
kaiserl. königl. Hofbuchdruckern und Buchhändlern.
1783.

Figure 4: Vega „log tables“ - 1783

On page XIX in the preface in Vega's logarithmic and trigonometric tables as well as in the first part of his lectures (p 412 – 425) we can find the following example, which gives an insight into Vega's method of calculating the logarithms.

The example shows the log(1+x) expressed by an infinite power series:

$\log(1+x) = Ax + Bx^2 + Cx^3 + Dx^4 + \ldots$.

and similar

$\log(1+y) = Ay + By^2 + Cy^3 + Dy^4 + \ldots$.

A, B, C, … are unknown constants and to compute them we set:

$(1 + x)^2 = 1 + y$

and due to the logarithms rules

$$2 \log(1 + x) = \log(1 + y).$$

If we substitute this in the series above, it results in:

$$2Ax + 2Bx^2 + 2Cx^3 + 2Dx^4 + \ldots = Ay + By^2 + Cy^3 + Dy^4 + \ldots .$$

From $(1 + x)^2 = 1 + 2x + x^2 = 1 + y$ we get for the powers of y the expressions:

$$y = 2x + x^2$$

$$y^2 = 4x^2 + 4x^3 + x^4$$

$$y^3 = 8x^3 + 12x^4 + 6x^5 + x^6$$

$$y^4 = 16x^4 + \dots .$$

If we substitute the powers of y above and take the difference, we get:

$$0 = (2A - 2A)\,x + (A + 4B - 2B)\,x^2 + (4B + 8C - 2C)\,x^3 +$$

$$+ (B + 12C + 16D - 2D)\,x^4 + (6C + 32D + 32E - 2E)\,x^5 + \dots .$$

So all coefficients have to be zero and these are the conditions that have to be fulfilled, and solving the linear system we obtain:

$$2A - 2A = 0 \qquad A = A$$

$$A + 4B - 2B = 0 \qquad B = -\frac{1}{2}A$$

$$4B + 8C - 2C = 0 \qquad C = +\frac{1}{3}A$$

$$B + 12C + 16D - 2D = 0 \qquad D = -\frac{1}{4}A$$

$$6C + 32D + 32E - 2E = 0 \qquad E = +\frac{1}{5}A .$$

log (1+x) is expressed by this series

$$= A\,x - \frac{1}{2}A\,x^2 + \frac{1}{3}A\,x^3 - \frac{1}{4}A\,x^4 + \dots .$$

Some further steps are explained in Vega’s lectures in a didactically excellent way and the necessary condition for convergence is mentioned as well.

That is an algebraic calculation which does not have anything to do with convergence at all. So the notion of convergence of a sequence was floating around during the 18^{th} century. It really came into focus properly in the 19^{th} century when Cauchy brought to mind what converge meant for a sequence.

On page 414 we find a remark about convergence of the series and on page 416 he wrote that we must expand 6 terms of the series to obtain an accuracy of 7 places.

2000 copies of this work were sold and therefore Vega published a second edition. He wrote the prologue of this edition during the War of the First Coalition in February 1797. It was impossible for him to find an editor for his books in Vienna and so he was forced to go abroad. This was the reason for publishing the second edition in Leipzig in 1797.

A lot of scientists praised this book. There was a notice in the "Allgemeinen Literaturzeitung" of Jena on February 17, 1798 and the mathematician Kästner spoke very highly of these tables in the "Göttinger Anzeiger" of October 21, 1797.

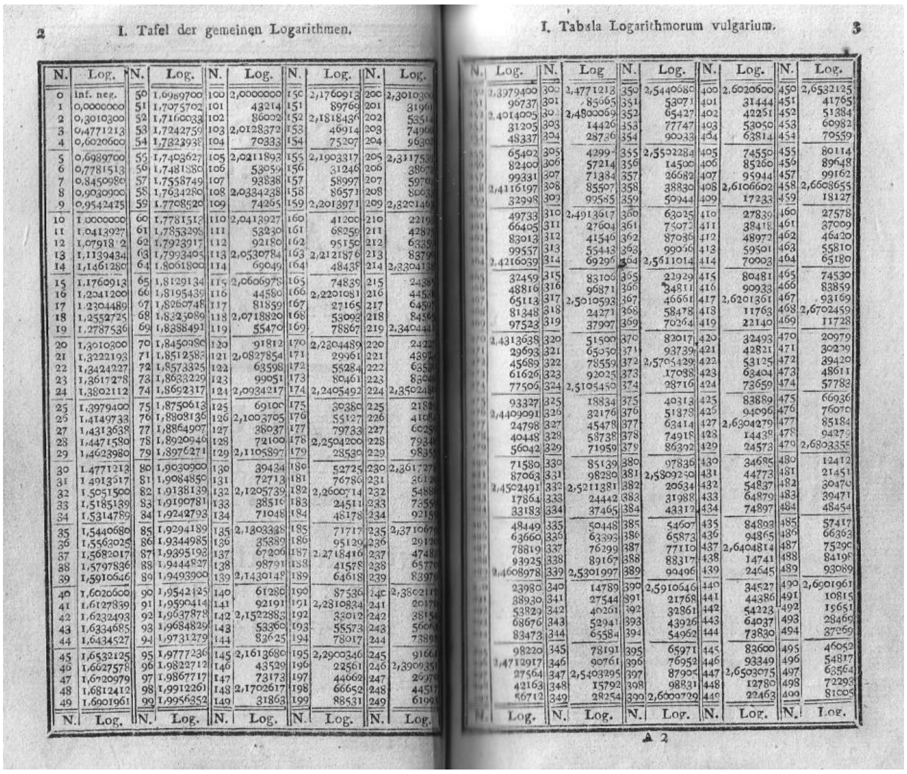
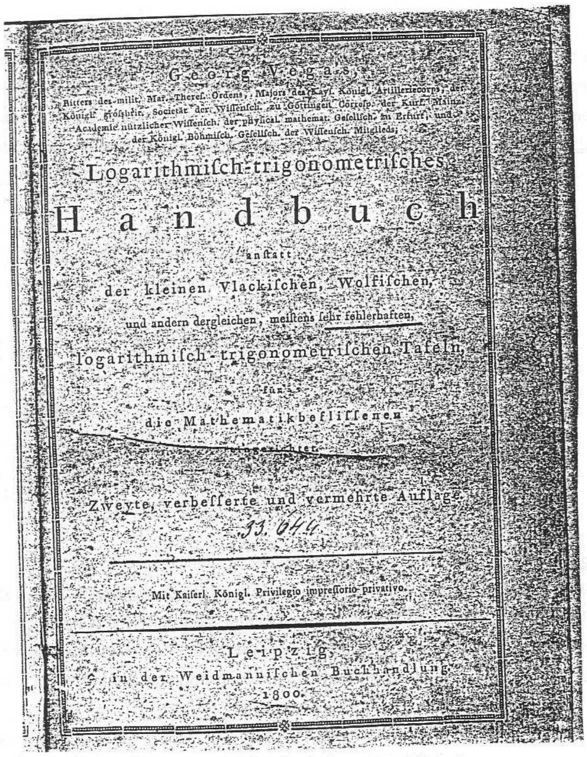


Figure 5: Vega „log tables" - page 2

However, between the publishing of these two editions Vega published three other books.

The "Logarithmisch-trigonometrisches Handbuch" was written in German and Latin, entitled "Manuale logarithmico trigonometricum matheseos etc."



Georg Vegas,

Ritters des milit. Mar. Theres. Ordens, Majors des Kayl. Königl. Artilleriecorps, der Königl. großbrit. Societät der Wissensch. zu Göttingen Corresp. der Kurf. Mainz. Academie nützlicher Wissensch. der physical. mathemat. Gesellsch. zu Erfurt, und der Königl. Böhmisch. Gesellsch. der Wissensch. Mitglieds;

Logarithmisch-trigonometrisches

Handbuch

anstatt

der kleinen Vlackischen, Wolfischen,

und andern dergleichen, meistens sehr fehlerhaften,

logarithmisch-trigonometrischen Tafeln,

für

die Mathematikbeflissenen

eingerichtet.

Zweyte, verbesserte und vermehrte Auflage.

Mit Kaiserl. Königl. Privilegio impressorio privativo.

Leipzig

in der Weidmannischen Buchhandlung

1800.

Figure 6: Vega "Logarithmisch-trigonometrisches Handbuch" - 1800

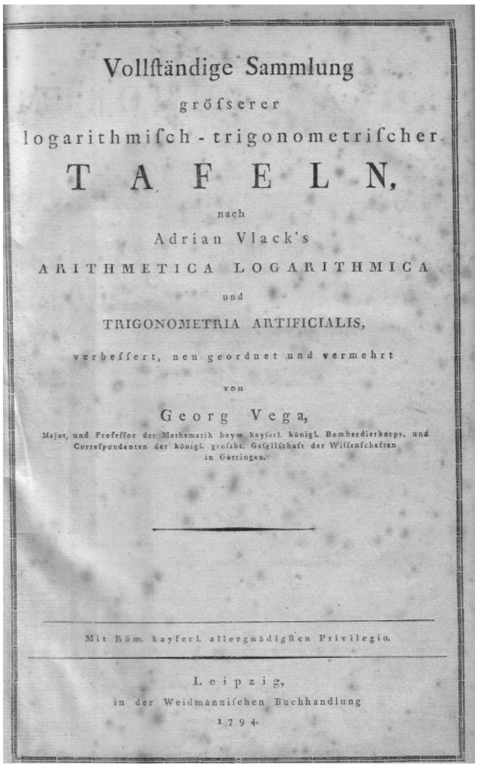
This work contains common logarithms with index and 7 decimal places of the natural numbers from 1 to 100 000 and the logarithms of the trigonometric functions.

In the prologue the theory of logarithms is explained in detail in Latin and German. In contrast to his first work Vega used fewer mathematical formulas. You can also find examples of applications for the tables. More than 80 editions were printed of this book and it was translated into English, French, Italian, Dutch, and Russian. The later editions were done by Hülsse, Bremiker and Tietjen.

In the meantime the tables of Vlacq were out of print and a lot of people were looking for new tables. Moreover, France changed all measures to decimal positions and the director of the "Bureau du cadastre" got the order to make new tables.

Perhaps 70 to 80 mathematicians calculated the "sine numbers" with 25 decimal places and their logarithms with 14 decimal places. The logarithms of the natural numbers from 1 to 10 000 were calculated with 19 decimal places and the numbers of 10 000 to 200 000 with 14 decimal places.

These tables would have needed 1 200 pages and due to the devaluation of banknotes they could never be printed. And so Vega's third tables were published at an appropriate time in 1794.



Vollständige Sammlung
grössérer
logarithmisch - trigonometrischer
T A F E L N,
nach
Adrian Vlack's
ARITHMETICA LOGARITHMICA
und
TRIGONOMETRIA ARTIFICIALIS,
verbessert, neu geordnet und vermehrt
von
Georg Vega,
Major, und Professor der Mathematik beym kayserl. königl. Bombardierkorps, und Correspondenten der königl. grossbr. Gesellschaft der Wissenschaften in Göttingen.

Mit Röm. kayserl. allergnädigsten Privilegio.

Leipzig,
in der Weidmannischen Buchhandlung
1794.

Figure 7: Vega „Thesaurus“ - 1794

This book was published for astronomers, navigators, professors and all lovers of mathematics who had to make exact calculations.

This work is based on Vlacq's "Arithmetica logarithmica seu logarithmorum chiliades centum .. (1628)" and "Trigonometria artificialis ... (1633)". Vega corrected wrong digits and added new tables. Together with mathematicians from the artillery corps in Vienna he verified the printed book. One of those mathematicians was Ignaz Lindner.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0. 000 0000 000	67	1. 826 0748 027	134	2. 127 1047 984	201	2. 303 1960 574	267	2. 426 5112 614
2	0. 301 0299 957	68	1. 832 5089 127	135	2. 130 3337 685	202	2. 305 3513 694	268	2. 428 1347 940
3	0. 477 1212 547	69	1. 838 8490 907	136	2. 133 5389 084	203	2. 307 4960 379	269	2. 429 7522 800
4	0. 602 0599 913	70	1. 845 0980 400	137	2. 136 7205 672	204	2. 309 6301 674	270	2. 431 3637 642
5	0. 698 9700 043	71	1. 851 2583 487	138	2. 139 8790 864	205	2. 311 7538 611	271	2. 432 9692 909
6	0. 778 1512 504	72	1. 857 3324 964	139	2. 143 0148 003	206	2. 313 8672 204	272	2. 434 5689 040
7	0. 845 0980 400	73	1. 863 3228 601	140	2. 146 1280 357	207	2. 315 9703 455	273	2. 436 1626 470
8	0. 903 0899 870	74	1. 869 2317 197	141	2. 149 2191 127	208	2. 318 0633 350	274	2. 437 7505 628
9	0. 954 2425 094	75	1. 875 0612 634	142	2. 152 2883 444	209	2. 320 1462 861	275	2. 439 3326 938
10	1. 000 0000 000	76	1. 880 8135 923	143	2. 155 3360 375	210	2. 322 2192 947	276	2. 440 9090 821
11	1. 041 3926 852	77	1. 886 4907 252	144	2. 158 3624 921	211	2. 324 2824 553	277	2. 442 4797 691
12	1. 079 1812 460	78	1. 892 0946 027	145	2. 161 3680 022	212	2. 326 3358 609	278	2. 444 0447 959
13	1. 113 9433 523	79	1. 897 6270 913	146	2. 164 3528 558	213	2. 328 3796 034	279	2. 445 6042 033
14	1. 146 1280 357	80	1. 903 0899 870	147	2. 167 3173 347	214	2. 330 4137 733	280	2. 447 1580 313
15	1. 176 0912 591	81	1. 908 4850 189	148	2. 170 2617 154	215	2. 332 4384 599	281	2. 448 7063 199

Figure 8: Vega „Thesaurus“ - first table

Here we can find the common logarithms of the natural numbers from 1 to 101 000 with ten decimal places together with the index. In the prologue many examples are given how to find the logarithm of a number.

On the first page there are the logarithms of the trigonometric functions for the range of 2 degrees with an interval of 1 second. The values of the further angles of the first quadrant are tabulated with an interval of 10 seconds together with the differences. You can also find many examples and the reasons for the exact calculation of parts of seconds which were needed for calculations of spherical triangles.

Furthermore, we can find other trigonometric tables in this book.

This "Thesaurus logarithmorum completus" has 713 pages and was very successful when it was published. The selling price was low as well.

Georg Simon Klügel (1739-1812) wrote in his mathematical dictionary that the title "Thesaurus logarithmorum completus" is legitimated by this book. In the "Astronomische Nachrichten" (astronomical news) of May 2, 1851 Carl Friedrich Gauss made a critical notice because there were a lot of wrong numbers on the last digit.

However, it was a character trait of Gauss to give a bad press. *“M. Max de Leber published at Vienna in 1897 a list of errata in Vega’s tables: it is, however, very incomplete.”* [1]

Nevertheless, nearly a hundred years after the first publication the mathematician Bremiker, who published the 82nd edition, said that the "Thesaurus" was still the best table with 10 decimal places. *“Vega’s ten-place tables were accurate enough for almost all practical applications for over 150 years, when they were eventually replaced by electronics.”*[2]

3 Leopold of Unterberger

Figure 9: Unterberger - Therese Rak (Wien 1902) “Ein österr. General…“

3.1 Biography

[1] H. Andoyer translated by Knott, Cargill Gilston in: Napier tercentenary memorial volume. p. 245, 246.(London 1915).

[2] Edward Sandifer, Why 140 Digits of Pi Matter in: Baron Juij Vega and His Times. p. 251.(Slovenije 2006).

He was born on October 27, 1734 in Strengberg in Lower-Austria. His father was a butcher and a landlord. He got his first lessons by his parents and later he attended the Catholic Grammar School in Seitenstetten and in Linz. In the Lyzeum in Linz the Jesuit priest Joseph Walcher filled him with enthusiasm for mathematics and in 1758 he entered the "Ingenieurkorps" as a cadet. During the Seven Years' War he was promoted lieutenant and senior lieutenant. 1770 he was awarded the professorship of mathematics in the "Artillerieschule".

The director of the artillery Kinsky ordered him to write a textbook for this school. He published his first book in Vienna 1774. (The title was "Anfangsgründe der Mathematik, zum Gebrauche der mathematischen Schule des k. k. Artilleriekorps".) He published the second part entitled "Geometrie" in the same year and tables of logarithms two years later.

Unterberger was the commanding officer in the "Bombardierkorps", which was founded in 1786. Two years later he entered the war together with Vega. After the siege of Belgrad he was promoted colonel.

During the War of the First Coalition he was in the battlefield again together with Vega. Due to the siege of Mannheim he wrote an attestation for the excellent performance of Vega. It was written in Mannheim on December 16, 1795.

Unterberger got further promotions. He died on February 9, 1818. 137 books were found in his estate and 4 of them were written by Vega. The value of all these books was said to be 6 Gulden.

3.2 Unterberger's tables

1777 Unterberger published the tables of logarithms
"Tafel der Sinusse, Tangenten und Secanten mit ihren Logarithmen**,** nebst den Logarithmen der natürlichen Zahlen von 1 bis 20 000".

Tafel
der
Sinusse, Tangenten
und
Sekanten,
mit ihren
Logarithmen,
nebst
den Logarithmen der natürlichen Zahlen
von 1. bis 20000.

WIEN,
gedruckt bey Johann Thomas Edlen v. Trattnern,
kaiserl. königl. Hofbuchdruckern und Buchhändlern.
1777.

Figure 10: Unterberger „Tables of Logarithms“ - 1777

To render mathematicians and geometricians a great service, he created these tables during his time as a professor in the "Feldartillerie-Korps" as well as a teacher of Archduke Maximilian. You can find the common logarithms of the natural numbers from 1 to 20 000 with seven decimal places and furthermore, you find logarithms of trigonometric functions. Unterberger did not use series for calculating logarithms because he used the method of Briggs with Geometric means.

The method of calculation can be seen in the following example.
It amounts to bracketing the numbers whose logarithms we desire between two numbers whose logarithms are known.

We’ll find the logarithms of 5:

$10^0 < 5 < 10^1$ $\qquad x_1 = \sqrt{10^0 10^1} = 10^{1/2}$

$0 < \log 5 < 1$ $\qquad \log x_1 = \frac{1}{2}$

$10^{1/2} < 5 < 10^1$ $\qquad x_2 = \sqrt{10^{1/2} 10^1} = 10^{1/4}$

$\frac{1}{2} < \log 5 < 1$ $\qquad \log x_2 = \frac{1}{4}$

This is continued as long as needed to achieve the required accuracy.

In 1617 Henry Briggs computed the first table to the base of 10. In the prologue Unterberger explained that Vlacq’s and Gardiner’s tables are ideally suited for astronomers but on the other hand they are inconvenient for simple geometric tasks. Furthermore, he mentioned that the price was too high.

In spite of Dominique François Rivard’s (1697 - 1778) tables were useful and exceptionally correct but despite several editions they were out of print. Together with Joseph Pichler (Oberleutnant k. k. Artilleriekorps) he extended Rivard’s tables and published a new edition. In his estate Rivard’s tables were found and the value of these tables was said to be 40 Kroner. (1 Gulden = 2 Kroner)

Logarithmen der Sinusse.

Sekn.	Minuten. 0′	1′	2′	3′	4′
1	46855749	64709047	67683602	69432534	70675918
2	49866049	64779665	67719347	69456462	70693901
3	51626961	64849154	67754800	69480259	70711810
4	52876349	64917548	67789965	69503926	70729646
5	53845449	64984882	67824849	69527465	70747408
6	54637261	65051188	67859454	69550878	70765099
7	55306729	65116497	67893786	69574164	70782717
8	55886649	65180838	67927848	69597327	70800264
9	56398174	65244239	67961645	69620366	70817741
10	56855749	65306729	67995182	69643284	70835148
11	57269676	65368332	68028461	69666082	70852485
12	57647561	65429074	68061488	69688760	70869753
13	57995182	65488977	68094265	69711321	70886953
14	58317029	65548066	68126796	69733765	70904085
15	58616661	65606361	68159086	69756094	70921149

Figure 11: Unterberger „Tables“ - first table

In the first table we see the common logarithms of the sines from 1 second to 30 minutes and the characteristic is not separated by a comma. We cannot find this table in Rivard’s tables, and Unterberger was able to calculate this table due to the absolute advantage for trigonometric calculations.

Moreover, we find

- Tables of the logarithms of sine, tangent and secans

0 Grad. Logarith. der Sinusse und Sekant. Grad. 89.

M.	Log. Sinus	Differ.	Log. Kosek.	Log. Sekn. 10.	D.	L. Kosin. 99.	M
0	0		unendlich	.0000000			60
1	64637261	3010300	135362739	.0000000			59
2	67647561	1760912	132352439	.0000001	1	.999999	58
3	69408473	1249387	130591527	.0000002	1	.999998	57
4	70657860	969100	129342140	.0000003	2	.999997	56
5	71626960	791811	128373040	.0000005	2	.999995	55
6	72418771	669468	127581229	.0000007	2	.999993	54
7	73088239	579918	126911761	.0000009	3	.999991	53
8	73668157	511524	126331843	.0000012	3	.999988	52
9	74179681	457574	125820319	.0000015	3	.999985	51
10	74637255	413926	125362745	.0000018	4	.999982	50
11	75051181	377884	124948819	.0000022	4	.999978	49
12	75429065	347619	124570935	.0000026	5	.999974	48

Figure 12 Unterberger „Tables“ - logarithms of sine and secant

Please note: in the cosine column there are 2 digits less due to saving space.

- Tables of the logarithms of natural numbers from 1 to 20000 and the differences

Tafel der Logarithmen.

N.	Logarith.	Differ.	N.	Logarith.	Differ.
1	00000000		41	16127839	
2	03010300	3010300	42	16232493	104654
3	04771213	1760913	43	16334685	102192
4	06020600	1249387	44	16434527	99842
5	06989700	969100	45	16532125	97598
6	07781513	791813	46	16627578	95453
7	08450980	669467	47	16720979	93401
8	09030900	579920	48	16812412	91433
9	09542425	511525	49	16901961	89549
10	10000000	457575	50	16989700	87739
		[illegible]			[illegible]

Figure 13: Unterberger „Tables“ - logarithms of natural numbers

4 Ignaz Lindner

4.1 Biography

Despite all the effort there could not be found any picture of Ignaz Lindner, who was born in Prague in 1777.

You cannot find his name in any encyclopedia but the author was able to find some personal information about him. In 1793 he joined the "1. Feldartillerie-Regiment" in Prague as a cannoneer and got a salary of 10 Gulden. 14 days later he was promoted to senior cannoneer. At that time he had an excellent knowledge of mathematics. He helped Vega to finish the second part of his lectures and for this work he wrote a lot of parts himself.

During the War of the First Coalition he entered the "Bombardierkorps" and he was listening to Vega's lectures. In 1795 he became a bombardier. Four years later on March 16, 1799 he was promoted again. By attending Vega's lectures he expanded his mathematical knowledge.

On February 11, 1802 he was promoted to pyrobolist and one year later he published his first mathematical book. The title of this work is written in the "Judicium delegatum militare mixtum" but due to the crabbed handwritten letters it is not possible to read it.

In 1805 he was promoted lieutenant and four years later he came to the "Hungarian Insurection”. As Vega's tables were out of print and other tables could not be sold due to the high price, there were not enough logarithmic tables in Austria. Therefore, Lindner decided to write a logarithmic handbook.

In the year 1812 he was promoted to senior lieutenant and two years later to captain, and in the same year he became a professor for mathematics in the "Bombardiercorps". Field marshal Colloredo ordered him to write a mathematical book for his pupils, which was published in Vienna in 1817.

Lindner got further promotions and he died as a colonel on May 25, 1835 in Vienna. He did not only know Vega's mathematical books but also the books of Unterberger and Pasquich's "opusculis statico mechanicis".

4.2 Lindner’s tables

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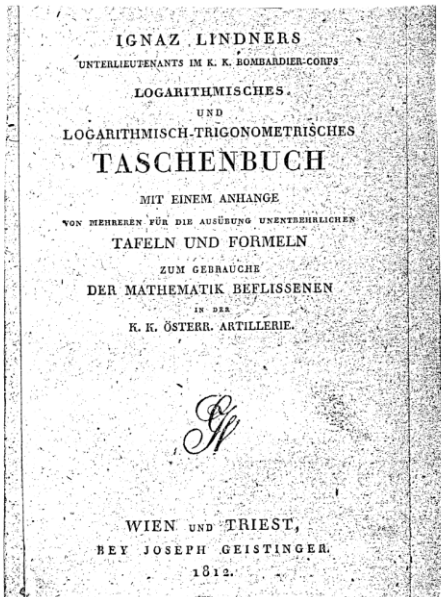

Figure 14: Lindner „log tables“ 1812

Corrections of 85 digits of Vega's tables are posted in the prologue and the theory of logarithms is explained. Lindner showed the calculation of logarithms by continued fractions. Certainly he knew Vega’s tables and infinite series to calculate logarithms but he did not mention it.
In contrast to Vega’s and Unterberger’s tables it explained that the logarithms of a composite number can be found by decadal supplementation.

N. 100. L. 00. Dif. 434. bis 299.

N.	0	1	2	3	4	5	6	7	8	9
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174
102	8600	9026	9451	9876	.0300	.0724	.1147	.1570	.1993	.2415
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616
104	7033	7451	7868	8284	8700	9116	9532	9947	.0361	.0775
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978
107	9384	9789	.0195	.0600	.1004	.1408	.1812	.2216	.2619	.3021
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028
109	7426	7825	8223	8620	9017	9414	9811	.0207	.0602	.0998

Figure 15: Lindner „log tables“ - logarithms of natural numbers

In the first table we find the logarithms of the natural numbers up to 10000. Please note that the similar first two digits are only printed once.

Logarithmen der Bogen, Sinus und Tangenten für die ersten 72 Secunden von $\frac{1}{10}$ zu $\frac{1}{10}$ Secunden.

o Grad — o Minuten

Sec.	Dec.	Logarith.	Sec.	Dec.	Logarith.	Sec.	Dec.	Logarith.	Sec.	Dec.	Logarith.
18	0	5.940847	22	5	6.037757	27	0	6.116939	31	5	6.183885
	1	5.943253		6	6.039683		1	6.118544		6	6.185262
	2	5.945646		7	6.041601		2	6.120144		7	6.186634
	3	5.948026		8	6.043510		3	6.121738		8	6.188002
	4	5.950393		9	6.045410		4	6.123325		9	6.189366
	5	5.952747	23	0	6.047303		5	6.124908	32	0	6.190725
	6	5.955088		1	6.049187		6	6.126484		1	6.192080
	7	5.957416		2	6.051063		7	6.128055		2	6.193431
	8	5.959733		3	6.052931		8	6.129620		3	6.194777
	9	5.962037		4	6.054791		9	6.131179		4	6.196120
19	0	5.964328		5	6.056643	28	0	6.132733		5	6.197458
	1	5.966608		6	6.058487		1	6.134281		6	6.198792
	2	5.968876		7	6.060323		2	6.135824		7	6.200123
	3	5.971132		8	6.062152		3	6.137361		8	6.201449
	4	5.973377		9	6.063973		4	6.138893		9	6.202771
	5	5.975609	24	0	6.065786		5	6.140420	33	0	6.204089
	6	5.977831		1	6.067592		6	6.141941		1	6.205403
	7	5.980041		2	6.069390		7	6.143457		2	6.206713
	8	5.982240		3	6.071181		8	6.144967		3	6.208019

Figure 16: Lindner „log tables“ - logarithms of trigonometric functions

The second table as well as the third table shows the logarithms of trigonometric functions.

In the appendix we find some more useful tables for the calculation of bowstrings, comparison of different scales of temperature, measure of length and weights and others, more.

Franz Pfaff, who was a student and friend of Lindner, copied Vega’s logarithms into this edition. 3000 copies of this book were printed and in the year 1831 Lindner published a second edition.

5 Johann Pasquich

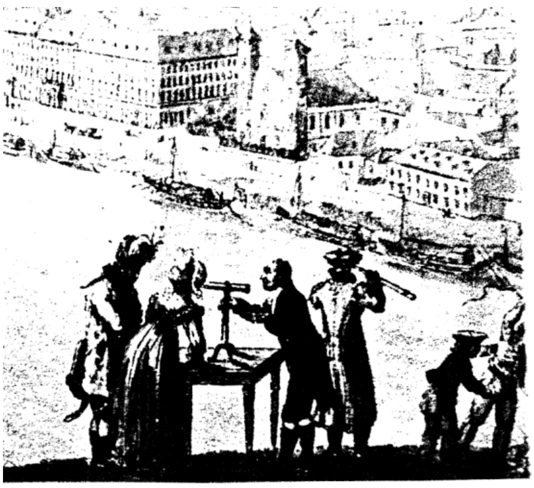

Figure 17: Picture museum in Budapest, Pasquich next to the telescope

5.1 Biography

Johann Pasquich was born in Zengg in Croatia in 1753. He attended the School of the Archducal Board in Senjska. In 1775 he was ordained as a priest. Later he was promoted to master of philosophy in 1781. In 1784 he published his first mathematical and astronomical work and became a doctor of philosophy.

Since 1784 he stayed at the University of Pest and he was an inspector of the observatory in Buda. From 1788 to 1797 he was a professor for advanced mathematics at the University of Pest.

During this time he published a mathematical work entitled "Unterricht in der mathematischen Analysis und Maschinenlehre" (lessons in mathematical analysis and mechanics). Georg of Vega wrote a very positive review of the second part of this book. He said: "This work exceeds all similar books ...".

In 1801 Pasquich wrote a letter to Vega with the request for publishing new tables of logarithms. The content of these tables should be a connection of the squares of the trigonometric values to their logarithms. But Vega died one year later and these tables were not calculated.

Later Pasquich worked at the observatory in Vienna and Gotha. In 1817 he also published tables of logarithms. He spent the last years of his life in Vienna, where he died on November 15, 1829.

5.2 Pasquich's tables

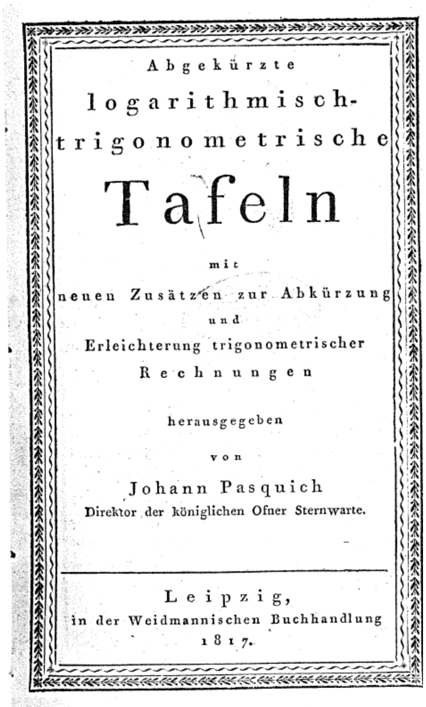
Abgekürzte
logarithmisch-
trigonometrische
Tafeln
mit
neuen Zusätzen zur Abkürzung
und
Erleichterung trigonometrischer
Rechnungen
herausgegeben
von
Johann Pasquich
Direktor der königlichen Ofner Sternwarte.
Leipzig,
in der Weidmannischen Buchhandlung
1817.

Figure 18: Pasquich „log tables" - 1817

In 1812 Gauss encouraged Pasquich to ask one of his friends to construct and publish tables. But Pasquich noticed that his friend could not fulfill this request or he would not be able to finish it quickly enough, and so he decided to provide these tables himself.

The book contains 3 tables:

- In the first table we see the common logarithms of the natural numbers up to 10000.

2 I. Tab. Logarithmorum

Num.	Log.	Num.	Log.	Num.	Log.	Num.	Log.
100	2.00000	145	2.16137	190	2.27875	235	2.37107
101	2.00432	146	2.16435	191	2.28103	236	2.37291
102	2.00860	147	2.16732	192	2.28330	237	2.37475
103	2.01284	148	2.17026	193	2.28556	238	2.37658
104	2.01703	149	2.17319	194	2.28780	239	2.37840
105	2.02119	150	2.17609	195	2.29003	240	2.38021
106	2.02531	151	2.17898	196	2.29226	241	2.38202
107	2.02938	152	2.18184	197	2.29447	242	2.38382
108	2.03342	153	2.18469	198	2.29667	243	2.38561
109	2.03743	154	2.18752	199	2.29885	244	2.38739
110	2.04139	155	2.19033	200	2.30103	245	2.38917
111	2.04532	156	2.19312	201	2.30320	246	2.39094
112	2.04922	157	2.19590	202	2.30535	247	2.39270
113	2.05308	158	2.19866	203	2.30750	248	2.39445
114	2.05690	159	2.20140	204	2.30963	249	2.39620

Figure 19: Pasquich „log tables“ - logarithms of natural numbers

- The second table shows the logarithms of trigonometric functions.

68 II. Tab. Logarithmorum vulg.

1. Gradus.

M.	L. Sin.	D. 1″	L. Cos.	D. 1″	L. Tng.	D. 1″	L. Cot.	M.
18	8.35578	9.22	9.99989	0.00	8.35590	9.22	11.64410	42
19	36132	9.10	89	2	36143	9.11	63857	41
20	36678	8.99	88	0	36689	8.99	63311	40
21	37217	8.88	88	0	37229	8.89	62771	39
22	37750	8.77	88	2	37762	8.78	62238	38
23	38276	8.67	87	0	38289	8.67	61711	37
24	38796	8.57	87	0.00	38809	8.57	61191	36
25	39310	8.47	87	2	39323	8.47	60677	35
26	39818	8.37	86	0	39832	8.37	60168	34
27	40320	8.27	86	0	40334	8.27	59666	33
28	40816	8.18	86	2	40830	8.18	59170	32
29	41307	8.08	85	0	41321	8.09	58679	31
30	41792	8.00	85	0.00	41807	8.00	58193	30
31	42272	7.91	85	2	42287	7.92	57713	29
32	42746	7.82	84	0	42762	7.83	57238	28
33	43216	7.74	84	0	43232	7.74	56769	27
34	43680	7.65	84	2	43696	7.66	56304	26
35	44139	7.58	83	0	44156	7.58	55844	25
36	44594	[illegible]	83	0.00	44611	[illegible]	55389	24

Sinuum, Cosin. etc. 69

1. Gradus.

Min.	Q. Sin.	Q. Cos.	Q. Tang.	Q. Cot.	Min.
18	0.00051	0.99949	0.00051	1941.8	42
19	53	47	53	1893.0	41
20	54	46	54	1845.9	40
21	56	44	56	1800.6	39
22	57	43	57	1756.9	38
23	58	42	59	1714.8	37
24	60	40	60	1674.2	36
25	62	38	62	1635.1	35
26	63	37	63	1597.2	34
27	65	35	65	1560.7	33
28	66	34	66	1525.4	32
29	68	32	68	1491.3	31
30	69	31	69	1458.4	30
31	71	29	71	1426.5	29
32	72	28	72	1395.6	28
33	74	26	74	1365.8	27
34	76	24	75	1336.8	26
35	78	22	77	1308.8	25
36	79	21	79	1281.7	24

Figure 20: Pasquich „log tables“ - logarithms of trigonometric functions

- The third table was taken over from the book of “Freiherrn v. Zach” according to Gauss’ calculations.

200 III. Tabula Gaussiana

A	B		C		A	B		C	
0.000	0.30103	50	0.30103	50	0.040	0.28149	47	0.32149	53
1	30053	50	153	50	1	102	48	202	52
2	0.30003	50	203	50	2	054	48	254	52
3	0.29953	50	253	50	3	0.28006	47	306	53
4	903	49	303	51	4	0.27959	48	359	52
5	854	50	354	50	5	911	47	411	53
6	804	50	404	50	6	864	47	464	53
7	754	49	454	51	7	817	48	517	52
8	705	50	505	50	8	769	47	569	53
9	655	49	555	51	9	722	47	622	53
0.010	606	50	606	50	0.050	675	47	675	53
1	556	49	656	51	1	628	47	728	53
2	507	49	707	51	2	581	47	781	53
3	458	49	758	51	3	534	47	834	53
4	409	50	809	50	4	487	47	887	53
5	359		859		5	440		940	

Figure 21: Pasquich „log tables“ - logarithms according to Gauss’ calculations

But Gauss expressed the wish that somebody would calculate a similar table with seven decimals with tenfold or hundredfold widening.
On October 4, 1817 there was a good review as well as a critical press in the newspaper “Göttingische Gelehrte Anzeigen”.

6 Conclusion

Finally it is worth mentioning that in the course of the last centuries several anniversaries have been celebrated in honour of Vega and Pasquich. Pasquich was well known as an astronomer in Austria and Hungary. Unterberger was admired as a military officer, but any tribute to his mathematical works is unknown. Lindner was not well known and celebrations are unknown.
Georg of Vega was the most important Austrian mathematician of the 18th century, but due to his European contemporaries like Leonhard Euler, Joseph Lagrange, Adrien-Marie Legendre, Pierre Laplace among others, he could not reach the position he would have deserved.

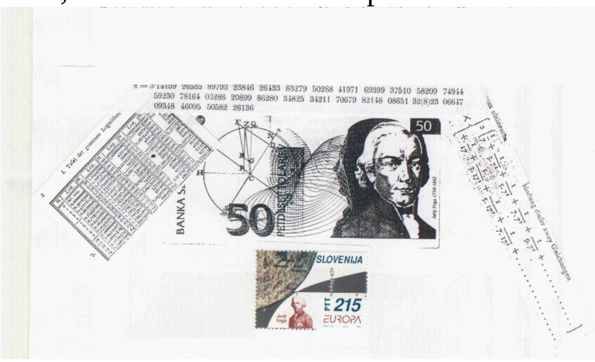

Figure 22: Vega - Slovenian 50 Tolar bill, stamp, tables

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