Napier's "Bones"

Michael R. Williams
Professor Emeritus
Department of Computer Science
University of Calgary
Calgary, Alberta
CANADA T2N 1N4

This meeting is gathered here to celebrate the 400^{th} anniversary of John Napier publishing his famous book *Descriptio* describing logarithms. While the majority of the speakers will concentrate on that subject, I will spend time filling in some background on some of his lesser known publications. In fact, almost all of my talk will concern his final publication known as *Rabdologiae*, but it seems only reasonable to also have a very quick look at his first (1593) publication *A Plaine Discovery of the whole Revelation of Saint John*.

PAX

AMOR

A PLAINE DIScouery of the whole Reuelation of Saint IOHN : set downe in two treatises : The one searching and prouing the true interpretation thereof: The other applying the same paraphrastically and Historically to the text.

SET FOORTH BY IOHN NAPEIR L. of Marchistoun younger.

WHEREVNTO ARE annexed certaine Oracles of SIBYLLA, agreeing with the Reuelation and other places of Scripture.

EDINBVRGH
PRINTED BY RObert Walde-graue, printer to the Kings Maiestie. 1593.
Cum Priuilegio Regali.

INFESTA malis.

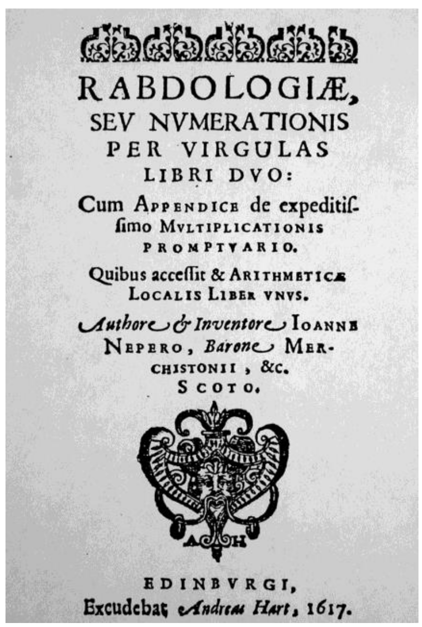
PACIS alumnus

Napier is, of course, best known for his mathematical studies but in his own time he was also well known for this book of Biblical scholarship – one of the earliest to be produced in Scotland. Today even most scholars have never seen a copy of this work and, if they have, will have found it difficult reading and full of references that they have difficulty understanding. We must remember that this was the age of the Protestant Reformation in Scotland and that it was a violent change which has had repercussions that still persist today. John Napier and his father were major figures in this social upheaval and this book was considered so important that it had several editions and was translated into several European languages. It is interesting that it is his only publication in English (all others were in Latin) because he wanted the common folk to be influenced by his arguments and, of course, Latin was only used by the better educated individuals. Even when he became known for his later mathematical work, he still considered this *Plane Discovery* as more important to the world than his invention of logarithms.

While John and his contemporaries might have thought highly of work, the modern reader will come away with the impression that it is nothing more than a rant against the Catholic Church in general and the Pope in particular. He attempts to use passages from the Biblical *Book of Revelation* to prove that all his Roman Catholic friends are destined to go straight to Hell and that the Pope is the Antichrist and the devil incarnate. To the modern reader it appears to be the ravings of a madman but it accurately reflects the strong feelings of the time and does provide some insight into the religious fervour of the late 16th Century Scotland. If he had published nothing else in his life, John Napier would, if anything, now have the reputation of being a raving lunatic.

The Rabdologiae

Now that we have had a quick glance at Napier's religious obsession, I will return to my main topic of his last publication.

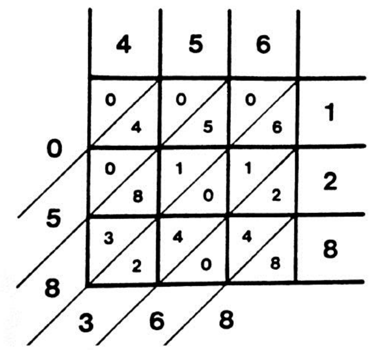

RABDOLOGIÆ,
SEV NVMERATIONIS
PER VIRGULAS
LIBRI DVO:
Cum APPENDICE de expeditiſ-ſimo MVLTIPLICATIONIS
PROMPTVARIO.
Quibus acceſſit & ARITHMETICÆ
LOCALIS LIBER VNVS.
Authore & Inventore IOANNE
NEPERO, Barone MER-CHISTONII, &c.
SCOTO.

EDINBVRGI,
Excudebat Andreas Hart, 1617.

This work appeared in the year of his death and 3 years after the *Descriptio*, his work on logarithms. In it he described three different methods of doing arithmetic: Napier's *virgulas* (better known as Napier's Bones); his *Multiplicationis Promptuario* (a more advanced version of his Bones), and his *Arithmeticae Localis* (Local Arithmetic or the use of a chess board and counters for doing arithmetic using binary notation). I will not describe Multiplicationis Promptuario simply because of time constraints and neither will I mention the Local Arithmetic because this is the topic for Steve Russ' contribution to this meeting.

The Origin of the Bones

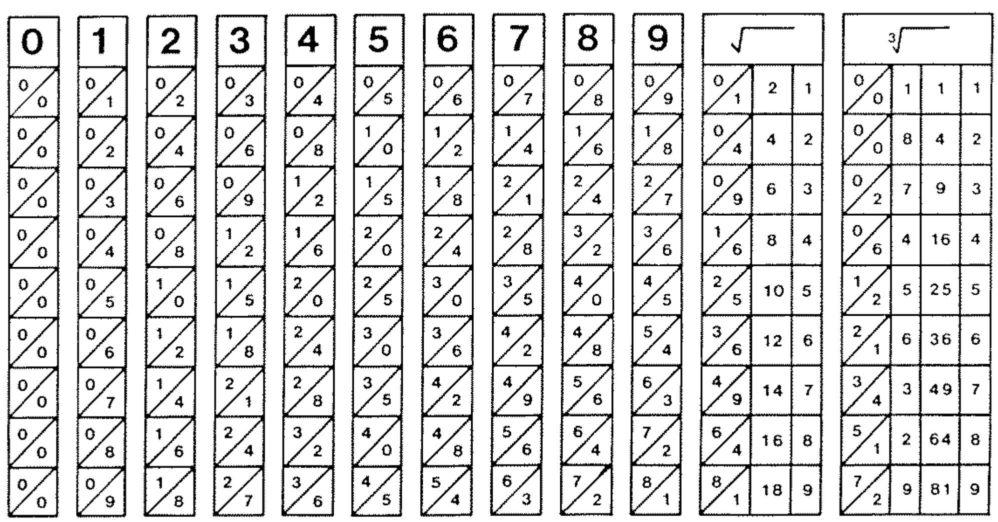
The concept of the Bones comes from an old form of doing multiplication known as the *Gelosia Method*. It is a method that was introduced into Italy in the 14th Century and the name comes from a form of Arabic window grating that was known as a Gelosia shutter in Italy. The Gelosia Method was used in the late middle ages by Arab, Persian and Chinese societies and it likely was originally developed in India.



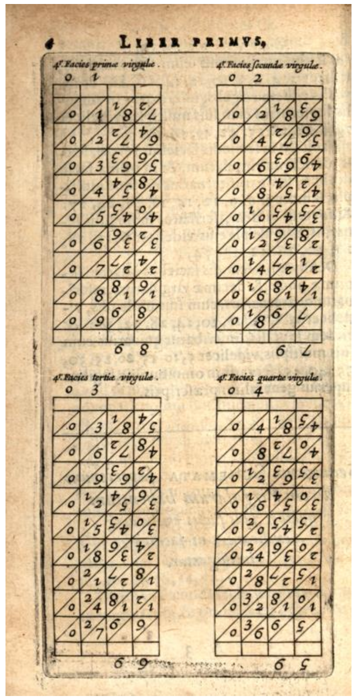
The gelosia method multiplication.

The two numbers to be multiplied (456 and 128 in this case) are written on two sides of a matrix-like grid and the products of the digits entered in the intersection of their rows and columns, the tens digit above a diagonal line and the units digit below. Beginning at the lower right, add up the digits in each diagonal and place the single digit sum at the end of the diagonal (any carries get added to the next diagonal). The product may then be easily read, beginning from the upper left, as 058368.

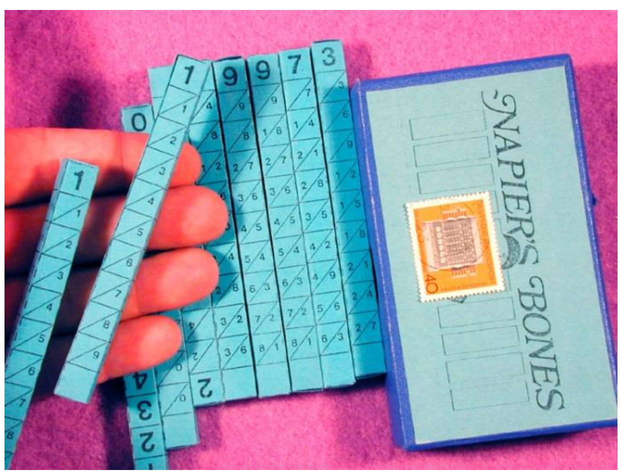
Napier's Bones are simply a collection of all possible columns of a Gelosia multiplication as shown below.



The bones noted as being for square and cube root calculations were part of Napier's original concept, but as they are difficult to use, and any description would be lengthy, I will not discuss them here. Napier envisaged the bones as rectangular rods with a different bone on each face but his illustration of them in *Rabdologiae* was an exploded view with each of the four sides being shown.

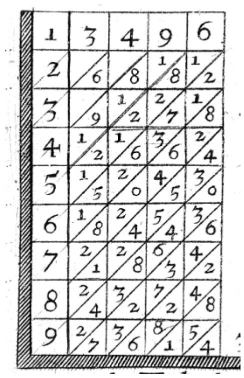


The Bones for 3, 4, 5, and 6 as Napier illustrated them.



A modern cardboard set of Bones

The use of the Bones is trivial and does not even require the knowledge of the multiplication table. For example, to multiply 3,496 by 6 (it is only possible to multiply by a single digit at any one step – a limitation that was overcome in the Multiplicationis Promptuario), you assemble the bones for 3,4,9 and 6 (together with an index bone for easy identification of the rows) as shown below.



To obtain the product, begin on the right hand side of the 6th row, add together the digits in each parallelogram spanning two bones (triangle on the left and right edges) as follows:

Units digit is 6

Tens digit is 3+4 = 7

Hundreds digit = 5 + 4 = 9

Thousands digit = 8 + 2 = 0 (carry the 1)

Ten Thousands digit = 1 + the carry = 2

Total Product = 20,976

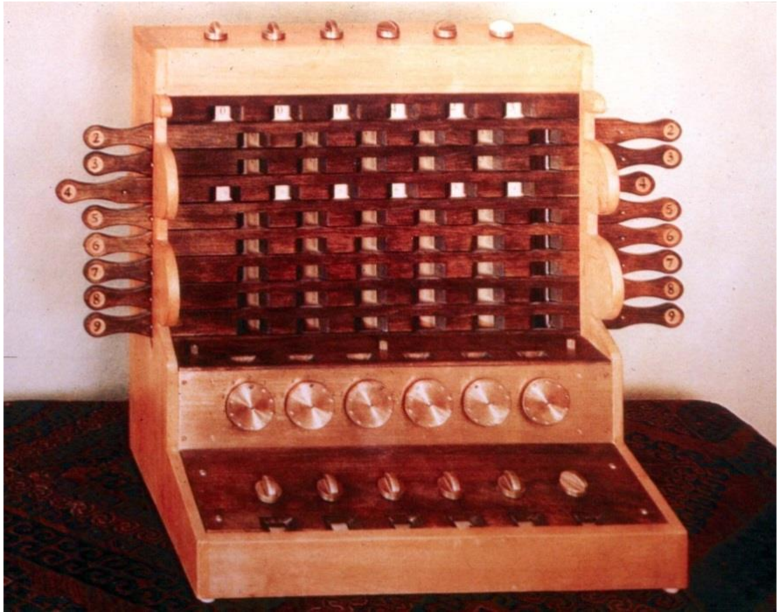
The Bones were an immediate hit and were copied and modified by many different individual to fit into their inventions or to make them easier to use – like many well intentioned efforts the results were often more difficult to use in practice.

William Schickard's Version of the Bones

Schickard was a professor of Hebrew, oriental languages, mathematics, astronomy and geography at Tubingen University – if that were not enough he also was a mechanic, painter, skilled engraver and a Protestant Minister in Tubingen. Schickard was a childhood friend of the great astronomer Kepler and the two of them often exchanged letters discussing mathematics, and other technical developments of their day including Napier's logarithms and his Bones. It was Kepler that first drew Schickard's attention to Napier and his works.

William Schickard (1592-1635)

In 1623 Schickard saw that it would be convenient to have some type of mechanical adding machine to add together the single digit products produced by the Bones when attempting multi-digit multiplications. After experimenting with a number of different configurations he created a unified machine with a set of Bones in the upper portion and an adding mechanism in the lower (the first such known to exist with an automatic carry mechanism). He wrapped cylinders with the diagrams of the bones (the set of bones for 0-9 on each cylinder) and these could be rotated to show the wanted digit through little windows in the upper portion of the device. Slides could be moved to open or close windows to show individual rows of bones (the slide for "4" is shown open in the illustration below).



Schickard's machine: photo of a modern reproduction.

Schickard was so pleased with this invention that he wrote to Kepler describing how it would help in astronomical calculations and that he had engaged a workman to make a copy for Kepler to use. Unfortunately he had to later write another letter telling Kepler that the workman's shop had burnt to the ground and everything had been lost. It is assumed that Schickard's own machine was also lost in the fire. We know the rough outline of the machine and what it did from diagrams Schickard drew on scraps of a letter that had been sent to Kepler. Kepler, in turn, had used these scraps as bookmarks in his table of logarithms and they were discovered there in the second half of the 20th Century when scholars were attempting to produce a definitive catalog of all Kepler's work.

Samuel Morland's (1625-1695) Bones

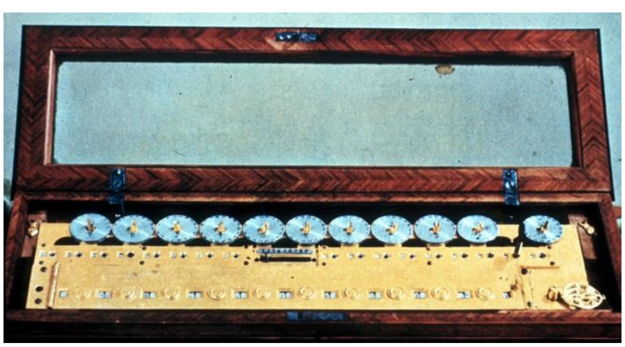
Samuel Morland led a full and interesting life (see Dickinson, H. W.; *Sir Samuel Morland, Diplomat and Inventor*, Cambridge, 1970). He was caught up in the turmoil surrounding the assumption of power by Oliver Cromwell and did not get a chance to attend university until he was much older than most of the other students. He eventually became a Fellow of Magdalene College in Cambridge, just in time to sign Samuel Pepys enrollment forms in 1650. He joined a diplomatic mission to Sweden, where at the court of Queen Christina he probably saw a copy of Pascal's adding machine. A later diplomatic mission to France and Italy gave him the opportunity to further study calculating machines as inventors such as Rene Grillet were active in such experiments in the French Court. Morland was able to maintain these contacts in later life because he married a French woman and made frequent trips back to the Continent. His simple calculating machines were not a great advance, and in his diary Samuel Pepys described them as being *very pretty but not very useful*.

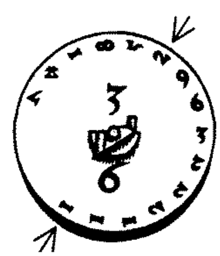
Morland, while acting as a diplomat for Cromwell, was also a spy for the exiled King Charles II. Upon Charles' return to the throne Morland was rewarded with honors and wealth, allowing him the leisure to invent several calculating aids (a mechanical sector, an adding machine, a circular disk form of Napier's bones) as well as barometers, speaking trumpets and water pumps.

Morland had advertised his adding machine in the London press, but only a few seem to have been purchased. Fortunately, a few have survived in museums. He presented a copy of his machine that uses a circular version of Napier's bones to the Duke of Tuscany, but no other examples are known to us.

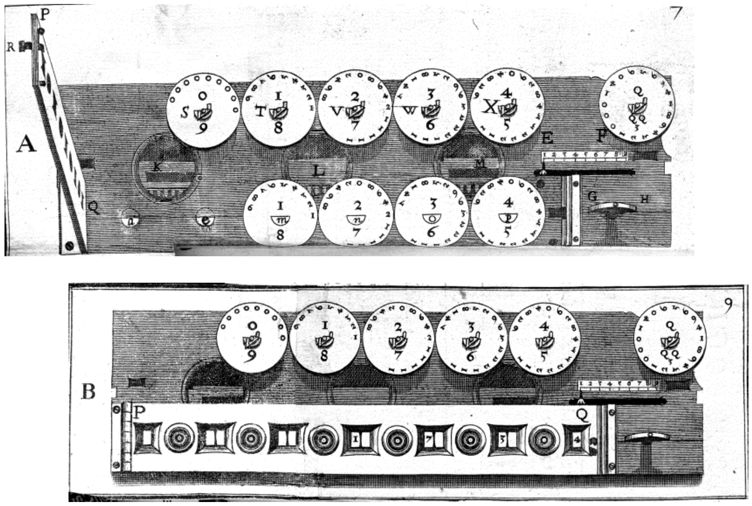
Samuel Morland the frontispiece from his 1673 book *The description and use of two arithmetick instruments*

His multiplying device has a set of Napier's bones engraved on brass circles with the units and tens digits on opposite ends of a diameter. By placing the appropriate circles into the machine and turning a key to the correct multiple, the digits showing through adjacent windows could be added together to create the product. The device was complex to produce and would have been enormously expensive to build when compared with the cost of a standard set of boxwood bones.





Morland's Bones based multiplication device and the circular bone for "3" (arrows show the fourth row of the "3" Bone – ie the product of 4 x 3 = 12)



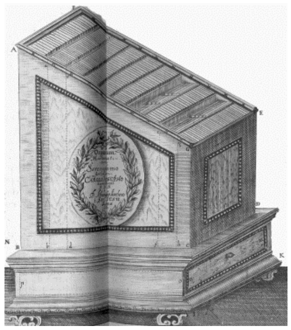
Morland’s machine as illustrated in his *The description and use of two arithmetick instruments*.

The top illustration shows the gate open with the circular bones on the top row being simply storage and the actual bones for multiplying the number (1234 in this case) having been mounted on the lower working pins. When the gate was closed (as in the second illustration) the small key (lower right) could be turned to rotate the disks to show any single digit multiple (the multiplying digit being noted on the linear scale in the middle at the far right). Note that the engraver made an error in that the number showing through the windows in the lower illustration is 1734 rather than 1234 it should be when the multiplying digit was still set to 1.

Gaspard Schott’s Bones (1668)

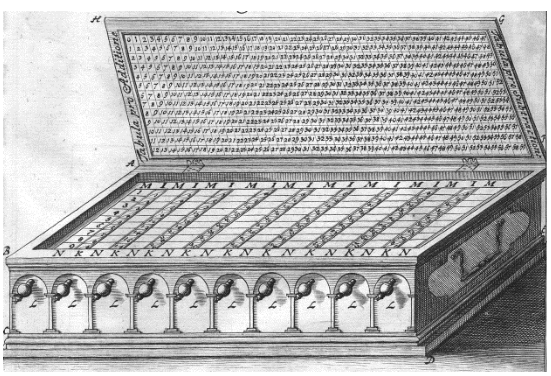
Gaspar Schott was a German Jesuit priest who attended Würzburg University and studied philosophy under Athanasius Kircher. Schott and Kircher fled Würzburg during the 1631 invasion by Sweden, Schott going to Palermo and Kircher, eventually, to Rome. Both men were in contact with Jesuit missionaries in various parts of the world, and Kircher, in particular, collected vast amounts of information on a wide variety of subjects. Schott was reunited with Kircher in 1652 but returned to Würzburg in 1655 with the intention of publishing Kircher’s notes and documents. He published eleven substantial volumes in the last eight years of his life. None of this material was new, but Schott combined many sources into one convenient volume. His works were widely admired and served to spread scientific information throughout the world.

In one of these great volumes, *Organum Mathematicum Libris IX Explicatum* published in 1668, Schott took the concept of Napier’s Bones and extended it to fields other than arithmetic. He based this work on a suggestion by Kircher of a collection of rods and tables that could be used for dealing with music, astronomy, surveying, astrology, dialing, the calendar, fortification and arithmetic. The collection fits into a box called the *Organum Mathematicum* which, while tempting to translate as *Mathematical Organ* is more appropriately termed *Mathematical Instrument*.



Schott's collection of Bones

As an addition to this remarkable collection, Schott also proposes a modification of Napier's bones in which a complete set is inscribed on cylinders. A number of such cylinders are mounted in a box such that an appropriate set of bones for any multi-digit number can be set up by simply rotating the handles of the cylinders. While this mechanism makes the selection of the individual bones easier and quicker than it would be for a normal set, it causes difficulty in their actual use due to the fact that the bones for adjacent digits are not close enough together to permit easy addition of the digits in the parallelograms. The only really useful result from having these cylindrical bones in a box was the provision of an addition table on the lid.

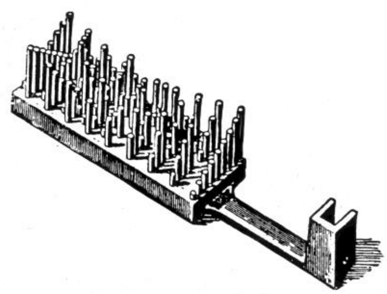


Schott's cylindrical Bones device as illustrated in Leupold, *Theatrum Arithmetico*, 1727, plate 5.

More Modern Versions of the Bones

Léon Bollée (1870-1913) was a French Automobile maker and inventor who apparently could turn his hand to inventing almost anything. He was intrigued by both the concept of Napier's Bones and how they might be incorporated into a mechanical multiplication mechanism. While not the first to construct a mechanical multiplier, he made a successful machine but it was so expensive that few were sold.

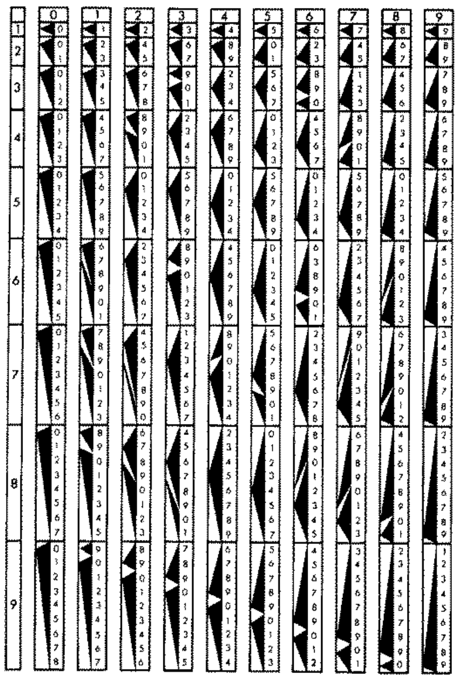
Léon Bollée using his mechanical calculating machine.



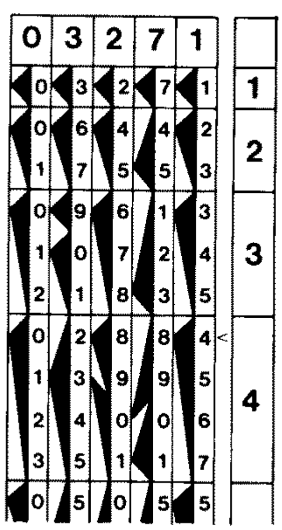
Bollée's mechanical Bones

Bollée converted Napier's bones into a mechanical device resembling a brass hair brush. The rods seen in the illustration are of different lengths, the sizes corresponding to the values of the digits in Napier's Bones (each Bone consists of 2 rows of rods across the unit, the first for the units digit and second for the tens). The complete set took 18 rows of rods of lengths between zero and nine units. When a number was to be multiplied, the machine moved the "hair brush" so that the correct set of two rows of rods would be pushed against a set of racks which would be moved by an amount equal to the length of the rod. While Bollée did not see great success with his invention, similar devices were later used successfully by others in production machines such as the Millionaire. Designed by Otto Steiger, a Swiss engineer, the Milllionaire sold over 5,000 machines in the early 20^{th} Century.

In the late 1880s two Frenchmen, **Edouard Lucas** (a science journalist) and **Henri Genaille** (a railway engineer) met at a conference when Genaille was speaking about how he had modified Napier’s Bones to eliminate the carrying of digits from one column (Bone) to the next. Lucas was inspired to find a similar solution for division and the two devices have since become known as Genaille Lucas Rulers. Genaille’s multiplication rulers are simply strips resembling Napier’s Bones but with dark triangular arrows that make it easy to simply read off a product without having to worry about carrying digits to the next column. An example is easier to see than a wordy explanation:



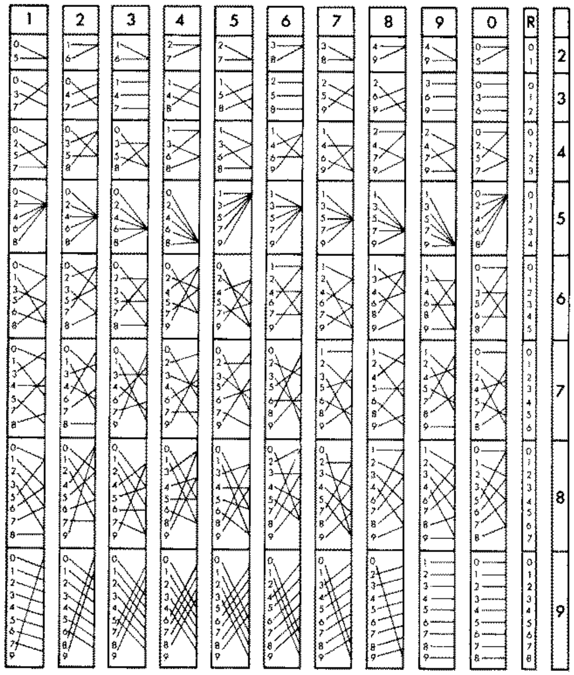
A complete set of Genaille/Lucas multiplication rulers



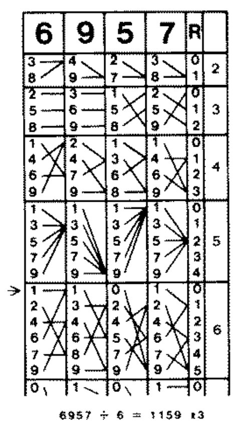
An example of 3,271 times 4

After laying out the Genaille rulers with a leading “0” ruler, starting from the uppermost digit in the fourth row (here marked with a small “<”) simply follow the dark arrows and read off the product from right to left (3,271 x 4 = 13,084).

The division rulers are equally easy to use.



Genaille/Lucas division rulers



An example of 6,957 divided by 6

After the rulers representing the number to be divided are assembled, together with a special remainder ruler on the right, one can read off the quotient by following the lines, beginning at the top left digit in the divisor row (6 – here marked by a small “>”) as follows: 1159 with 3 reminder.

These two sets of rulers are deceptive in their simplicity but are very difficult to devise if you are tempted to create a pair for yourself.

The final version of Napier’s Bones I wish to consider is one that was incorporated into a computer. In the late 1950s IBM decided to produce a small, relatively inexpensive, transistorized digital computer that would appeal to small engineering firms and academic institutions. The result was the IBM 1620. The cost was kept to a minimum by eliminating anything that was particularly expensive to design and produce – things such as the arithmetic unit. As a consequence this computer was always known as a *C.A.D.E.T.* machine which stood for *Can’t Add Doesn’t Even Try*. Rather than incorporating an expensive set of arithmetic circuits this machine used the first 400 decimal digits of its memory to store a set of tables (essentially a set of Napier’s Bones) so that the machine could look up the digits of any arithmetical operation and obtain the result – a slow, but economical, process. It was possible for programmers to change the base of any arithmetic operation (to, say, binary) by modifying the tables, performing the arithmetic operation, and then changing it back again (the operating system used the tables too, so any change to the table had to be immediately reversed or the machine would crash).

An IBM 1620 with the author at the console, circa 1961.

Napier's Real Bones

The last topic worth mentioning is that nobody knows where Napier's actual bones are resting. There is a sign in St. Cuthbert's Church in Edinburgh (on Lothian Road, just off Princes Street) that claims they are buried in a crypt below the church. This claim makes some sence because John Napier was known to be associated with this parish. However reports by two different individuals, both claiming to have been at his funeral, indicate two entirely different parish churches in Edinburgh. One of these was indeed St. Cuthbert's but, as the original church was modified and extended several times and eventually rebuilt completely in the 19th century, nobody can actually verify that Napier's earthly bones were preserved in their original crypt.

St. Cuthbert's Church Yard, Edinburgh