Logarithms – a Journey of their Tables to all over the World [1]

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1 Introduction

These days, nobody expects anything really new on the subject of logarithms. Surprisingly, however, there is still something to be written, something to be reviewed or something to be looked at again with fresh eyes. The history of logarithms has been described and dealt with on many occasions and is very clear. Nevertheless there are unnecessary discussions from time to time about who invented logarithms. Undeniably, a 7-place logarithmic table called "*Mirifici Logarithmorum Canonis Descriptio*" was published in 1614 for the very first time by *John Napier* (1550 – 1617).

6 years later "*Aritmetische und Geometrische Progreß Tabulen / sambt gründlichem Unterricht / wie solche nützlich in allerley Rechnungen zugebrauchen / und verstanden werden soll* " (translation see [3]) was published by *Jost Bürgi* (1552 - 1632) – also known as Joost or Jobst Byrgius or Byrg. He published logarithms to 8 places; however this table failed to catch on because there were no instructions for its use included as mentioned. In the eyes of some of today's contemporaries *Bürgi* is regarded as the inventor of logarithms, because he had been calculating his logarithms years before Napier, but he only published them on the insistence of *Johannes Kepler* (1571 – 1630). There is no information on the time it took *John Napier* to finish calculating his logarithms but in those times it would have been a matter of many years.

The theories of the Babylonians (about 1600 B.C.), *Euclid* (365 – 300 B.C.), and *Archimedes* (287 – 212 B.C.) had already begun to move in the logarithmic direction. Their theories were based on breaking calculations down to a lower level – for example simplifying a multiplication by using an addition - had paved the way to logarithms.

After *Nicolas Chuquet* (1445/1455 – 1487/1488) had documented the principle of a logarithm table by means of the comparison of tables of an arithmetic series, and a geometric series with a common

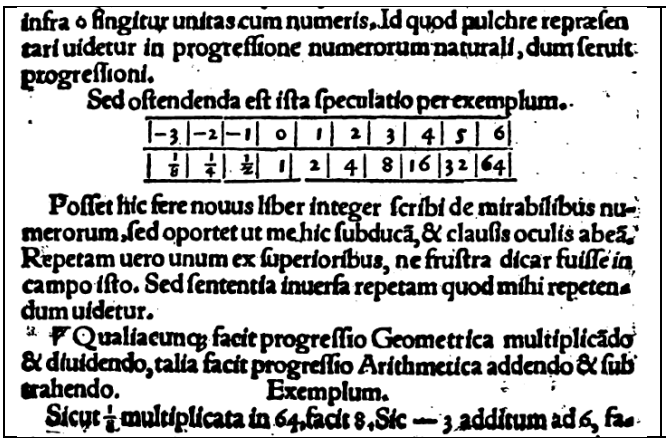
[1] Presented at a Joint Meeting of the James Clerk Maxwell Society and the British Society for the History of Mathematics (BSHM) in celebration of the 400th Anniversary of the publication of John Napier's *Mirifici Logarithmorum Canonis Descriptio.* Friday 4th April 2014 at Clerk Maxwell House

[2] This article I dedicate to Thomas Wyman who served as the first Oughtred * Society president from 1997 to 2007. Tom died March 17, 2014 at the age of 86 in Palo Alto, USA. He was one of the driving forces for spreading the idea of slide rule collecting from USA to all over the world. Tom published more than 80 papers on slide rules and logarithms.

* William Oughtred (1574 – 1660) developed the first slide rule around 1620

[3] *Arithmetical and geometrical progress tables / including instructions/ how to utilize and understand such tables; see Kathleen Clark at http://www.springer.com/fr/book/9781493931606*

ratio of 3 (see [4]) , *Michael Stifel* (1487 – 1567) in 1544 added negative numerical values to this approach in his "Arithmetica Integra", using geometric series with a common ratio of 2 – see figure 1.



infra o fingitur unitas cum numeris. Id quod pulchre repræſen tari uidetur in progreſſione numerorum naturali, dum ſeruit progreſſioni.

Sed oſtendenda eſt iſta ſpeculatio per exemplum.

−3	−2	−1	0	1	2	3	4	5	6
1/8	1/4	1/2	1	2	4	8	16	32	64

Poſſet hic fere nouus liber integer ſcribi de mirabilibus numerorum, ſed oportet ut me hic ſubducã, & clauſis oculis abeã. Repetam uero unum ex ſuperioribus, ne fruſtra dicar fuiſſe in campo iſto. Sed ſententia inuerſa repetenda uidetur.

¶ Qualiacunq; facit progreſſio Geometrica multiplicãdo & diuidendo, talia facit progreſſio Arithmetica addendo & ſubtrahendo. Exemplum.

Sicut 1/8 multiplicata in 64, facit 8, Sic — 3 additum ad 6, fa-

In the upper arithmetic progression each term after the first is found by adding a constant = 1 to the previous one. Those are the logarithms/exponents of the numeri.

In the lower geometric progression each term after the first is found by multiplying the previous one by a constant = 2. Those are the numeri.

Figure 1: Michael Stifel: Arithmetica Integra; Nürnberg 1544 – folio 249 [5]

Menso Folkerts [6] commented: "One can understand this table as a logarithmic table for $y = \log_2 x$ (for ten values of x such that $1/8 < x < 64$ and $-3 < y < 6$); however, it considers only integral y. On this Stifel remarks that one could write a whole new book about the wonderful properties of these numbers; but he must restrict himself and go by it with closed eyes.

Afterwords Stifel works through several examples. They show that Stifel fully grasped the correspondence between the arithmetic sequence of the exponents and the geometrical sequence of the powers and thus made a crucial contribution to the basis idea of logarithmic calculations. The first example is: 'Just as 1/8 multiplied by 64 yields 8, so if -3 is added to 6 there result is 3. 1/8 has the exponent -3 however, just as 6 is the exponent of 64 and 3 the exponent of 8 (Incidentally, the term 'exponent' was introduced, to our present knowledge, by Stifel).
With Stifel's extension of the arithmetic sequence to negative numbers, the breakthrough was made in the pre-history of logarithms. This was not enough for practical calculations, however: for that it was necessary to reduce the incremental width of the geometrical sequence in a way that any numbers which one wanted to multiply could be found in the sequence.
This only happened after *Simon Stevin (1548/49 – 1620)* had introduced decimal fractions at the end of the 16th century and had shown how arithmetic was done with them in the 'De Thiende' (1585).
The first usable logarithmic tables were calculated, independently of each other, by *Jost Bürgi*, printed in Prague 1620, and *John Napier*, printed in Edinburgh 1614. It is very likely that both were influenced by the passage just mentioned in Stifel's 'Arithmetica Integra'. Thus *Micheal Stifel* earns a solid place in the prehistory of the logarithms and so of the slide rule. "..and can be regarded as the inventor of the logarithmic principle and has brought it to the final quality.

All the following tablemakers were "only" the inventors of methods for calculating logarithms.

Logarithms were and are presently being used all over the world to facilitate calculations.

The French astronomer and mathematician *Pierre Simon Laplace* (1749 – 1827) once said:
"By shortening the labours Logarithms doubled the life of the astronomer".

[4] Erwin Voellmy: Jost Bürgi und die Logarithmen; Birkhäuser Verlag Basel, 2. Auflage 1974

[5] Link to Arithmetica Integra: https://archive.org/stream/bub_gb_fndPsRv08R0C#page/n6/mode/1up

[6] Menso Folkerts: Michael Stifel: in 7th International Meeting of Collectors of Slide Rules and Calculating Machines, page 35-36, (Editors: J. Konrad-Klein, K. Kühn, H. Petzold) Munich IM 2001

Following the appearance of the first table of logarithms in 1614, tables containing either the common (base 10) logarithms by *Henry Briggs* (1561 – 1630) from 1617 or the *Napier* logarithms quickly became more widespread, whereby the *Briggs* version proved to be more practical in terms of application and has therefore asserted itself. This was particularly due to the efforts of the Dutchman *Adriaan Vlacq* (1600 – 1667), who from 1628 onwards perfected the idea of selling tables with common logarithms. The first German edition of the *Vlacq* tables "*Tabellen der Sinuum, Tangentium und Secantium wie auch der Logarithmorum vor die Sinubus Tangentibus und die Zahlen von 1 bis 10000.*." was published in Amsterdam in 1673 by *Joan von Ravesteyn.*

These tables were not, however, the first ones ever to be published in German. In 1631 *Johannes Faulhaber* (1580 - 1635) had already published his "*Zehenttausent Logarithmi, der absolut oder ledigen Zahlen von 1 biß auff 10000. Nach Herrn Johannis Neperi Baronis Merchstenij Arth und invention, welche Henricus Briggius illustriert/ und Adrianus Vlacq augiert, gerichtet* " (see [7]) as a supplement to his Ingenieurs Schul (Engineers School) dated 1630. *Faulhaber* himself said in this publication "*Aus diesem erklärten Bericht kann man nun mehr verstehen den Ursprung oder Geburt der Logarithmorum, welches ein solche herrliche schöne Kunst / so mit Worten nicht auszureden ist. / Dann ist es nicht ein wunderbarliche Invention, daß in währendem Proceß das Extrahieren ins Halbieren verwandelt wird ?.....*" [8].

This work can be described as the first German mathematical textbook containing a logarithmic table and also including logarithms in the calculations with numerous examples taken from all areas of the art of numeration at that time.

Some years before, in 1618 *Benjaminus Ursinus* (1587 - 1633/4) published in Coloniae/Kölln near Berlin (now a part of Berlin itself) the first table of logarithms published on German/continental European ground. This table was written in Latin, and contained the Napier logarithms reduced to 5 places.

1.1 Yearly occurrence

The graph below shows the timeline of the publication of some 3000 tables of logarithms from 1614 to the last century. Most of the tables appeared and were used in the 19th and 20th century accompanying the rapid development of new technologies. Logarithms were intensively taught in schools in order to keep up with the requirements of industry.

Tables of logarithms appeared in 4 phases:

1. from 1614 to ca. 1680 initial phase with up to 8 tables in 1624
2. from 1680 to ca. 1780 a constant phase with up to 9 tables in 1742
3. from 1780 to ca. 1880 a strong yearly increase of publications with max. 26 in 1873
4. from 1880 to 1987 with an initial drecrease but catching up to 32 in 1960

Most of the documented tables were published in Europe as can be seen from the next table.

[7] *Ten thousand logarithms of numbers from 1 to 10000. Brought together like John Napier Laird of Merchiston´s, invention, illustrated (calculated) like Henry Briggs / and published by Adriaan Vlacq.*

[8] *In this book the origin or birth of logarithms will be understood as a wonderful art which can not be described by words. Isn´t it an admirable invention to replace extracting the square root by halving ?...*

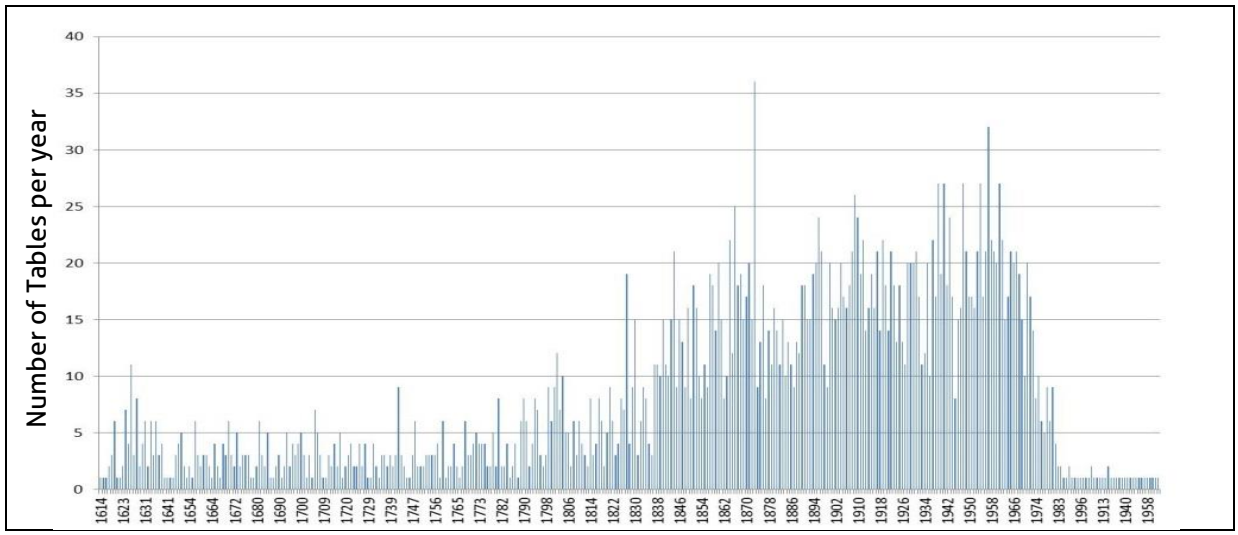


Figure 2: Number of published tables of logarithms per year

1.2 Cities of publishers

From the more than 3000 tables of logarithms which have been documented in the Collectanea [9] the table below includes some of the major cities of publishers. As can be seen those are mainly continental European cities - but from other continents as well.

For easier reading some of the names of the cities were transferred to English. This does not mean that those tables were published in English.

[9] Klaus Kühn: Collectanea de Logarithmis 2014

Table 1: Some cities where Tables of Logarithms have been published

Aberdeen	Edinburgh	Leiden	Regensburg
Altona	Eisenberg	Leipzig	Reichenberg
Amsterdam	Elberfeld	Leningrad	Roanne
AnnArbor	Erlangen	Liebenwerda	Rom
Arnhem	Essen	Liege	Rotterdam
Ashland	Estland	Lille	s´Gravenhage
Augsburg	Florence	Lissabon	Sagan
Avignon	Frankfurt	London	Salzburg
Bamberg	Friedenburg	Lübeck	Samedan
Barcelona	Geneve	Ludwigsburg	Sarajevo
Bergen	Genua	Lyon	Schwäbisch Gmünd
Berlin	Giessen	Madrid	Scranton
Bern	Glasgow	Magdeburg	Seville
Bielefeld	Glogau	Mainz	Shanghai
Bologna	Gotha	Mallorca	St. Malo
Bonn	Götheborg	Malmö	St. Petersburg
Bordeaux	Gouda	Mannheim	Stein bei Nürnberg
Boston	Graz	Marburg	Stockholm
Bratislava	Greifswald	Middleburg	Strassburg
Braunschweig	Groningen	Milano	Stuttgart
Breda	Gummersbach	Modena	Sulzbach
Bremen	Haarlem	Montreal	Tokyo
Bromberg	Copenhagen	Moscow	Toledo
Brünn	Halle	München	Torino
Bruxelles	Hamburg	Münster	Trento
Bulak	Hannover	Napoli	Triest
Cambridge	Harlingen	Neunkirchen	Tübingen
Campaniae	Heerbrugg	New Haven	Turin
Cape Town	Heidelberg	New York	Tyrnau
Capelli	Helmstedt	Nürnberg	Ulm
Chicago	Helsingfors	Oelsnitz	Upsala
Cirencester	Helsinki	Ohio	Valentiae
Cleveland	Herrmannstadt	Osnabrück	Vannes
Coburg	Hildesheim	Oxford	Venezia
Cologne	Ingolstadt	Paderborn	Verona
Coloniae (an der Spree)	Innsbruck	Padova	Vienna
Danzig	Iserlohn	Palermo	Warnemünde
Darmstadt	Ithaca	Paris	Warsaw
den Haag	Jena	Peking	Washington
Dessau	Karlsruhe	Pesaro	Weinheim
Deventer	Kempten	Philadelphia	Williamsburg
Dieppe	Kiel	Pisa	Wilmette
Dresden	Königsberg	Potsdam	Winterthur
Dublin	Kristiana	Prag	Wittenberg
Düsseldorf		Quebec	Wroclaw
	Zürich	Zagreb	Würzburg

This article will describe the journey of logarithmic tables to all over the world, starting with their route from Edinburgh to the European continent.

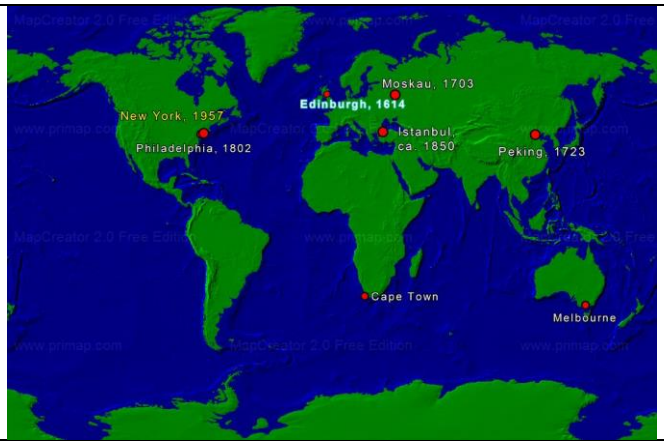


Figure 3: Journey of logarithmic tables to some major cities all over the world

2 From Edinburgh to the European Continent

2.1 In general

What follows is a tabular summary of the first tables of logarithms published by the calculators and authors in the early 17th century:

Year	Author/City	Title
1614 +1619	John Napier (1550 – 1617) Edinburgh	*Mirifici Logarithmorum Canonis Descriptio (1614)* *Mirifici Logarithmorum Canonis Constructio (1619)*
1617	Henry Briggs (1561 – 1630) London	*Logarithmorum Chilias Prima*
1619	John Speidell (1600 - 1634) London	*New logarithmes the first invention whereof, was, by the honourable Lo. Iohn Nepair, Baron of Marchiston, and printed at Edinburg in Scotland, anno 1614, in whose vse was and is required the knowledge of algebraicall addition and substraction, according to + and - : these being extracted from and out of them (they being first ouer seene, corrected, and amended) require not at all any skill in algebra, or cossike numbers, but may be vsed by euery one that can onely adde and substract ... / by Iohn [Spe]idell ... , [London] : ... to bee [solde at his] dwelling house in the Fields ...,* *(remark: first log nat of Numeri)*
1620	Jost Bürgi (1552 – 1632) Prague	*Aritmetische und Geometrische Progreß Tabulen – (calculated by Bürgi)*
1620	Edmund Gunter (1581 - 1626) London	*Canon triangulorum, sive tabulæ sinuum et tangentium artificialium*

1624	Johannes Kepler (1571 - 1630) Marburg	*Mathematici chilias logarithmorum ad totidem numeros rotundas – (calculated by Kepler)*
1625	Edmund Wingate (1596-1656) Paris	*Arithmetique logarithmique, or La Construction & Usage des Tables Logarithmetiques*
1628	Adriaan Vlacq (1600 - 1667) Gouda	*Arithmetica Logarithmica….ab Unitate ad 100.000…Sinuum, Tangentium, Secantium…*
1633	Nathaniel Roe (1596-1656) London	*Tabulæ logarithmicæ, or two tables of logarithmes*

Most of the early tables were published in England. Their spreading to the continent was mainly initiated by *Adriaan Vlacq*, a Dutch bookdealer. He had asked *Ezechiel de Decker* to complete the tables with missing content which were published by *Henry Briggs´* Arithmetica Logarithmica in 1624. After *de Decker* had done so in his *"Nieuwe Telkonst.."* in 1626, *Adriaan Vlacq* started later in 1636 a pocket size series of editions of *"Tabulae Sinuum, Tangentium, et Secantium, et Logarithmi Sinuum, Tangentium et Numerorum, ab unitate ad 10,000; Gouda"* [10]

This series was continued until 1768 (and with slight changes until 1821) in several languages - even in Chinese.

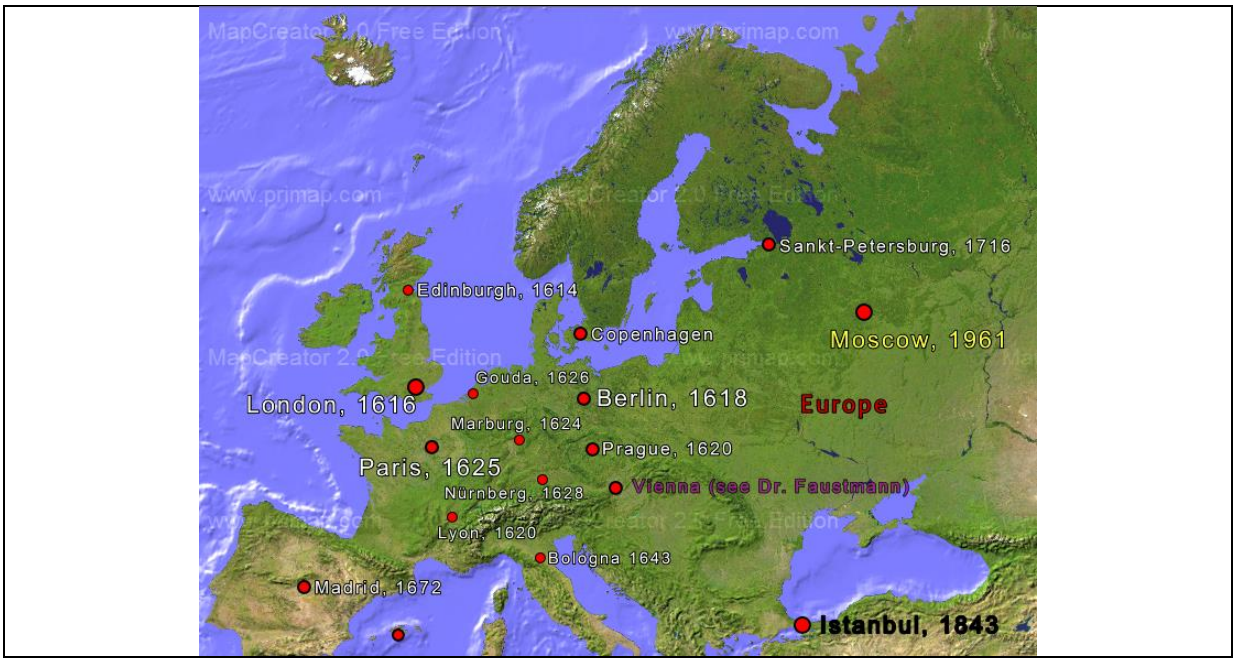


Figure 4: Cities where early (and later) Tables of Logarithms were published in Europe

[10] Otto E. van Poelje: Adriaen Vlacq and Ezechiel de Decker: Dutch contributors to the Early Tables of Briggsian Logarithms; Journal of the Oughtred Society 14 (2005) 30 - 40

2.2 From Scotland to Russia

In 1698 the Russian *Tsar Peter the Great* visited Scotland, looking for scientists to work for him in Russia.

Several of them took the chance and started a new carrer in Russia, mainly in Moscow and later in St. Petersburg. One of them was the young mathematician *Henry Fargwarson (Farquharson; Farkeson)* from Marischal College in Aberdeen - pupil of Professor *George Liddel*. *Fargwarson* was invited by *Tsar Peter* to Moscow in 1701 to direct the School of Mathematics (and Navigation), which was relocated to St. Petersburg in 1715 as the Naval Academy [11].

Henry Fargwarson (1675? - 1739) edited the first Russian Table of Logarithms together with *S. Gwynne* and *L. Magnitzkii* in 1703 in Moscow, followed by the second edition in 1716 also in Moscow (which was offered on ebay in October 2013) [12].

Figure 5: Title page of the 2nd edition 1716; R-ТАБЛИЦА ЛОГАРИФМОВ 1716 МАГНИЦКИЙ-эпоха Петра -R
1st edition 1703; translated by teachers of the marine academy

Trying to identify the source of the logarithms in that Russian table, it turned out that the edition of *Adriaan Vlaq´s* "Tabulae Sinuum, Tangentium, et Secantium, et Logarithmi Sinuum, Tangentium et Numerorum, ab unitate ad 10,000; Gouda" from 1670 looks pretty similar and the Russian title is even a translation from the Latin original.

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[11] For more detailed information see: The Caledonian Connection by Dimitri Fedosov; http://www.abdn.ac.uk/riiss/publications/centre-for-scottish-studies-publications-90.php

Or George Vernadsky: A History of Russia, Volume 5; Yale University Press, 1969 - 531 pages

[12] Demidov, Sergey; Moscow: Private Communication (2014) with hint to A.P. Yushkevich – Istoriya matematiki v Rossii do 1917 goda (History of Mathematics in Russia before 1917)

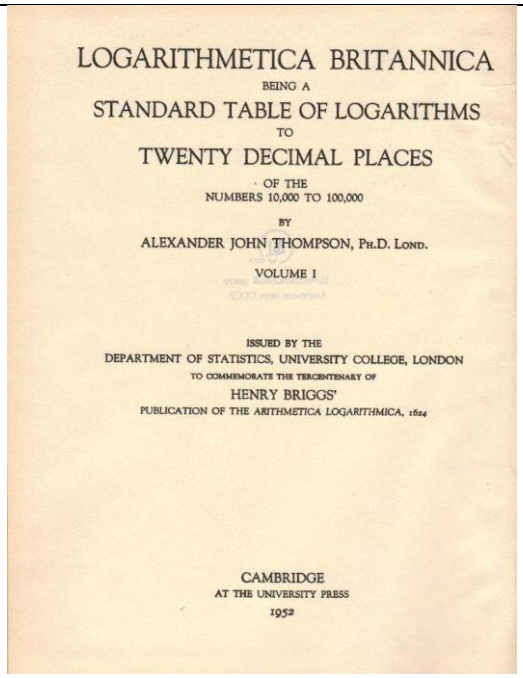
This Russian table from 1716 reflects the pretty intense connection between Scotland and Russia at that time.

70 Grad.

Minut.	Sinus	Tang.	Secans	Log.Sin.	L
30	94264.15	282391.29	299574.43	9.9743466	10
29	94254.43	282130.45	299328.56	9.9743018	10
28	94244.71	281870.03	299083.12	9.9742570	10
27	94234.98	281610.04	298838.11	9.9742122	10
26	94225.24	281350.48	298593.52	9.9741673	10
25	94215.50	281091.34	298349.36	9.9741224	10
24	94205.75	280832.63	298105.63	9.9740774	10
23	94195.99	280574.33	297862.31	9.9740324	10
22	94186.22	280316.46	297619.42	9.9739873	10
21	94176.44	280059.01	297376.95	9.9739422	10
20	94166.65	279801.98	297134.90	9.9738971	10

Figure 6: Pages for 70 degrees - Russian left – A. Vlaq Tabulae Sinuum, Tangentium, et Secantium, et Logarithmi Sinuum, Tangentium et Numerorum, ab unitate ad 10,000; Gouda 1670 right

In the 20th century there was another transfer of logarithms from the UK to Russia. The 1952 edition of the Logarithmica Britannica calculated by *Alexander John Thompson* (1855 – 1963??) was published in Moscow in 1961 (and probably again in 1972). These logarithms to twenty decimal places seemed to provide the accuracy which was needed for the Russian space program and other scientific works.



LOGARITHMETICA BRITANNICA

BEING A

STANDARD TABLE OF LOGARITHMS

TO

TWENTY DECIMAL PLACES

OF THE

NUMBERS 10,000 TO 100,000

BY

ALEXANDER JOHN THOMPSON, PH.D. LOND.

VOLUME I

ISSUED BY THE

DEPARTMENT OF STATISTICS, UNIVERSITY COLLEGE, LONDON

TO COMMEMORATE THE TERCENTENARY OF

HENRY BRIGGS'

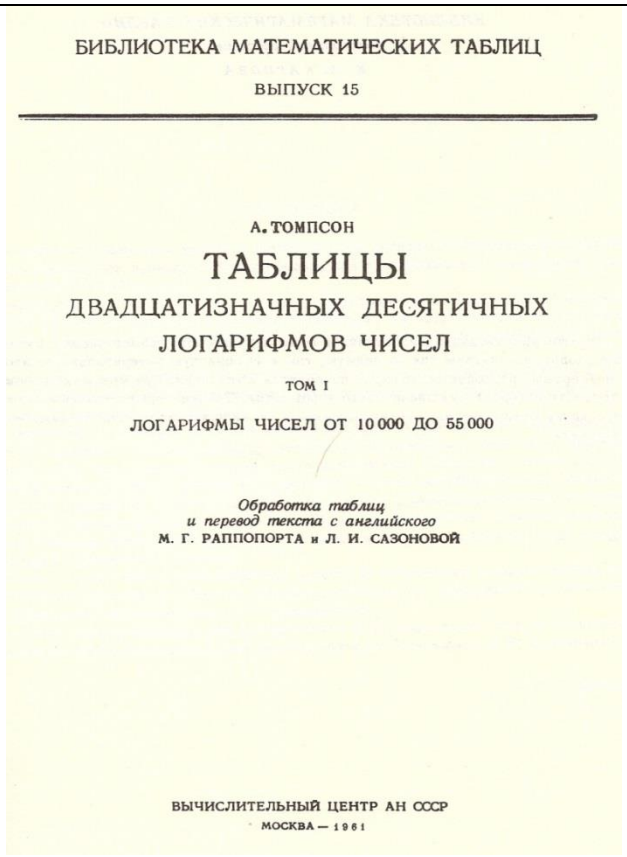
PUBLICATION OF THE *ARITHMETICA LOGARITHMICA*, 1624

CAMBRIDGE

AT THE UNIVERSITY PRESS

1952

Figure 7: Logarithmetica Britannica 1952 – volume 1 title



БИБЛИОТЕКА МАТЕМАТИЧЕСКИХ ТАБЛИЦ

ВЫПУСК 15

А. ТОМПСОН

ТАБЛИЦЫ

ДВАДЦАТИЗНАЧНЫХ ДЕСЯТИЧНЫХ

ЛОГАРИФМОВ ЧИСЕЛ

ТОМ I

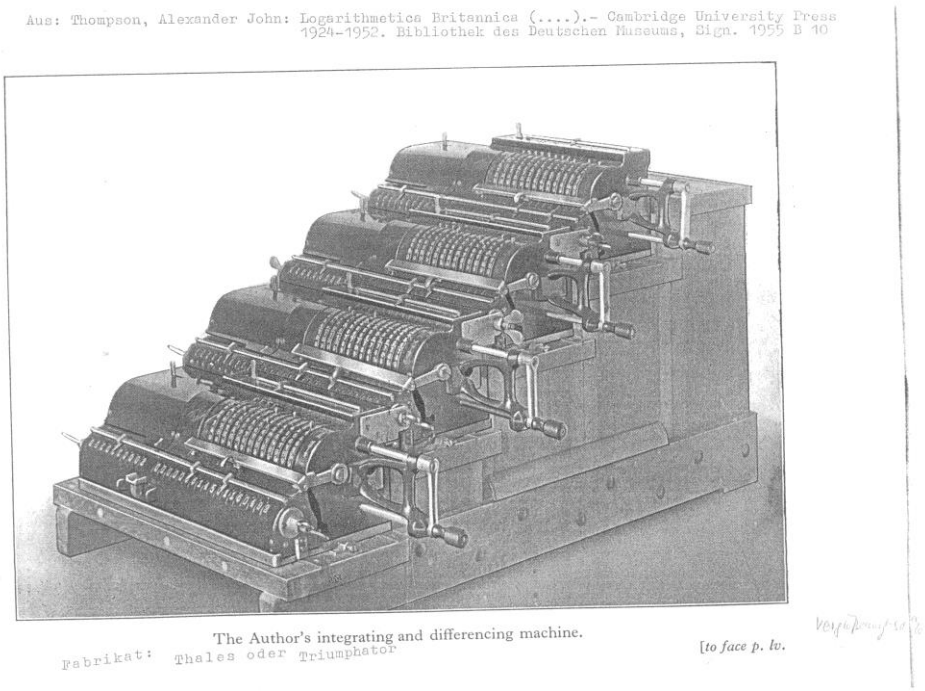
ЛОГАРИФМЫ ЧИСЕЛ ОТ 10 000 ДО 55 000

Обработка таблиц
и перевод текста с английского
М. Г. РАППОПОРТА и Л. И. САЗОНОВОЙ

ВЫЧИСЛИТЕЛЬНЫЙ ЦЕНТР АН СССР
МОСКВА — 1961

Figure 8: Logarithmetica Britannica – Russian version Moscow 1961 - volume 1 title

Below is a picture of the calculating machine which was used by *A.J. Thompson* for the calculation of the twenty decimal logarithms. [13]



Aus: Thompson, Alexander John: Logarithmetica Britannica (....).- Cambridge University Press 1924-1952. Bibliothek des Deutschen Museums, Sign. 1955 B 10

The Author's integrating and differencing machine.

Fabrikat: Thales oder Triumphator

[*to face p. lv.*

Figure 9: *A.J. Thompson´s* calculating machine was a combination of 4 Thales/Triumphator machines

[13] Stephan Weiss: http://www.mechrech.info/publikat/DMThomp.pdf

3 From United Kingdom to the Arabic Countries

Unfortunately the table of logarithms, two pages 0f which are shown below, has no title or date. Some handwritten annotations, though, give some hints on the timing and the local area of use.

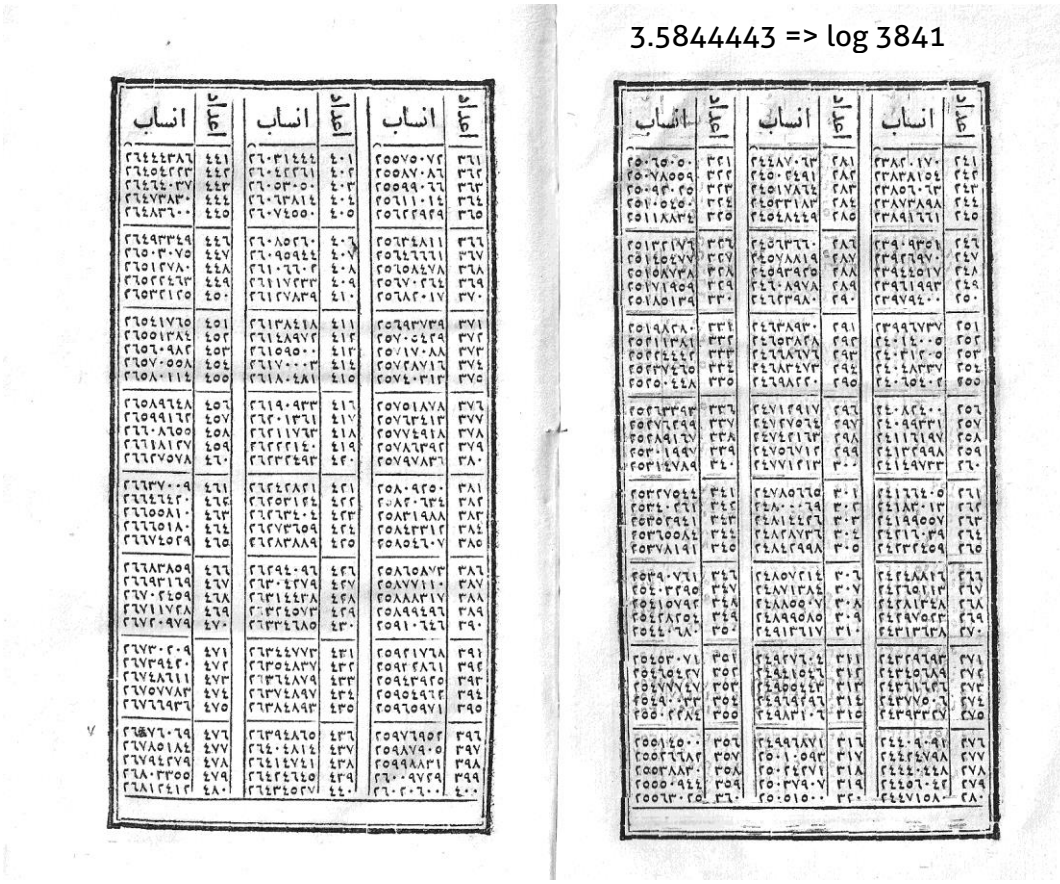


Figure 10: British Library Copy (no title); Annotated: „Acquired 1843 Forwarded to a friend in 1861 in Constantinople (Istanbul)" Zifers are East Arabic (Georges Ifrah, P. 518)

After this paper was presented in Edinburgh, Prof. *Alex D.D. Craik* [14] mentioned in the discussion that *Edward Sang* (1805 born in Scotland – 1890) was working in Constantinopel (Turkey) from 1843 to 1854, where he established engineering schools, and planned railways and ironworks. Before he left for Turkey, he was involved in editing logarithmic and trigonometrical tables. As stated by Rev. *John Smythe Memes* (1795 - 1858) in the introduction of *Robert Shortrede´s* (1800 – 1868) table of logarithms to seven digits of decimals from 1844, *Edward Sang* had calculated 3 tables of logarithms (1. Logarithms to Numbers; 2. Numbers to Logarithms; 3. Logarithmic Sines and Tangents to Every Second of the Circumference). 7 digit common logarithms to numbers as well as trigonometric logarithms to 7 digits are part of this "Arabic" Table of Common Logarithms. Thinking that *Edward Sang* was the editor/mentor of this "Arabic" table is probably not far from the truth.

Under the reign of *Ludwig XIV, Mehmed Effendi* (nickname "twenty-eight chalebi") was the ambassador of Turkey in France, when he met with *Jacques Cassini (1677 in France – 1756 in France)*, the director of the observatory in Paris. *Cassini* presented him the unpublished "Tables de Astronomique" from his father *Domenico Cassini (1625 in Italy – 1712 in France)*[15]. That was the

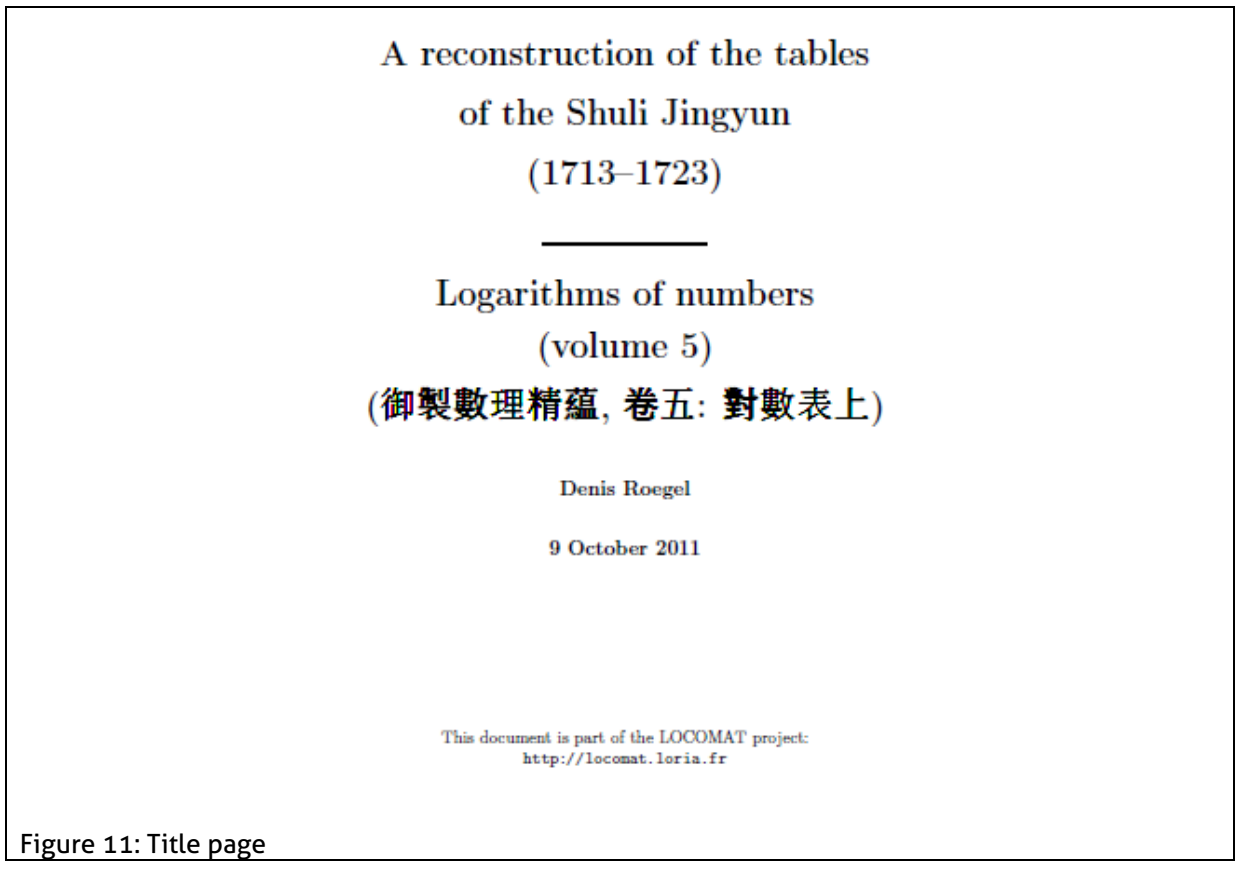
[14] Alex D.D. Craik: The logarithmic tables of Edward Sang and his daughters; Historia Mathemetica 30 (2003) 47- 84

[15] Salih Mourad: Introduction of Logarithms into Turkey (p 139 -144) in Cargill Gilston Knott: Napier – Tercentenary Memorial Volume, The Royal Society of Edinburgh, 1915
and: http://www.bilimtarihi.org/OBA/2007-8-2-4.htm

starting point for the introduction of logarithms to Turkey in 1714. The tables were rendered into Turkish in 1765 by *Ismail Effendi (1730 – 1791?)*, mathematician and Professor of Geometry at the Naval College in Istanbul. There is no clear evidence that those tables were pure Tables of Logarithms like the "Arabic" table mentioned above.

4 From Europe to the East - China

"The missionary *Mu Ni-ho* sometimes before 1660 introduced logarithms. In 1713 Adrian Vlack´s logarithmic tables to 11 (probably including the characteristic) places were reprinted" writes *Florian Cajori* in his History of Mathematics[16] from 1919. *Denis Roegel* [17] did an incredible amount of admirable reproductive work on logarithms (not only) from China and he states that "Logarithms were actually first introduced in China in 1653 by *Nikolaus Smogulecki* (1610–1656), a Jesuit missionary and his pupil *Xue Fengzuo* who adapted them. *Smogulecki*'s work was probably also based on that of *AdriaanVlacq*. There may have been other smaller tables of logarithms, but at this point, our knowledge on these tables is still very scarce.....These (10 digit) logarithms were most certainly copied from *Vlacq*'s Arithmetica Logarithmica (1628). The values seem to be identical with those printed in the 3-volume set, although the latter are laid out differently." *Roegel* reconstructed those tables in 2011 and has introduced to Chinese and Japanese tables of logarithms also in 2011 [18].



A reconstruction of the tables
of the Shuli Jingyun
(1713–1723)

Logarithms of numbers
(volume 5)
(御製數理精蘊, 卷五: 對數表上)

Denis Roegel

9 October 2011

This document is part of the LOCOMAT project:
http://locomat.loria.fr

Figure 11: Title page

[16] Florian Cajori: History of Mathematics; The Macmillan Company New York 1919

[17] Denis Roegel: http://locomat.loria.fr/locomat/reconstructed.html

[18] Denis Roegel: http://locomat.loria.fr/vlacq-chinese/clogdoc.pdf

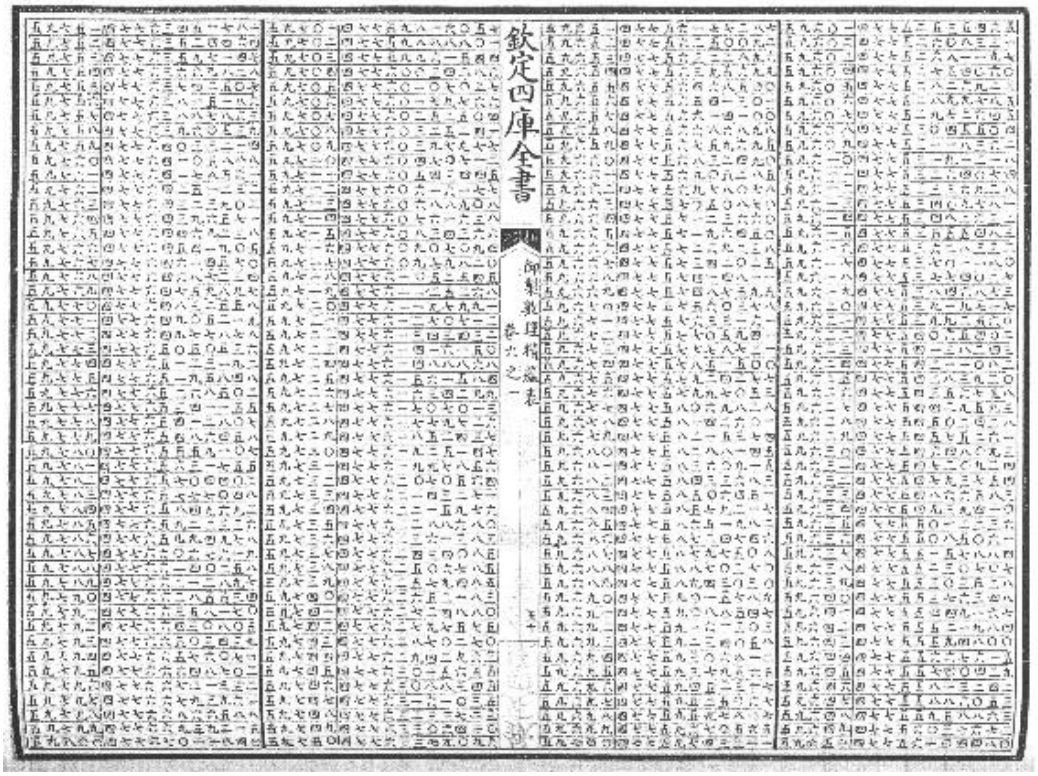


Figure 12: "Figure 7: Excerpt of the Siku Quanshu version of the table of logarithms of numbers (reduced printing from ([19])"

Centuries later, probably around 1974, Tables of Logarithms from *Friedrich Gustav Gauss* (1829 – 1915) [20], who had published more than 100 editions for schools, appeared on the Chinese market (Text in Chinese, numbers in European type) – as seen below.

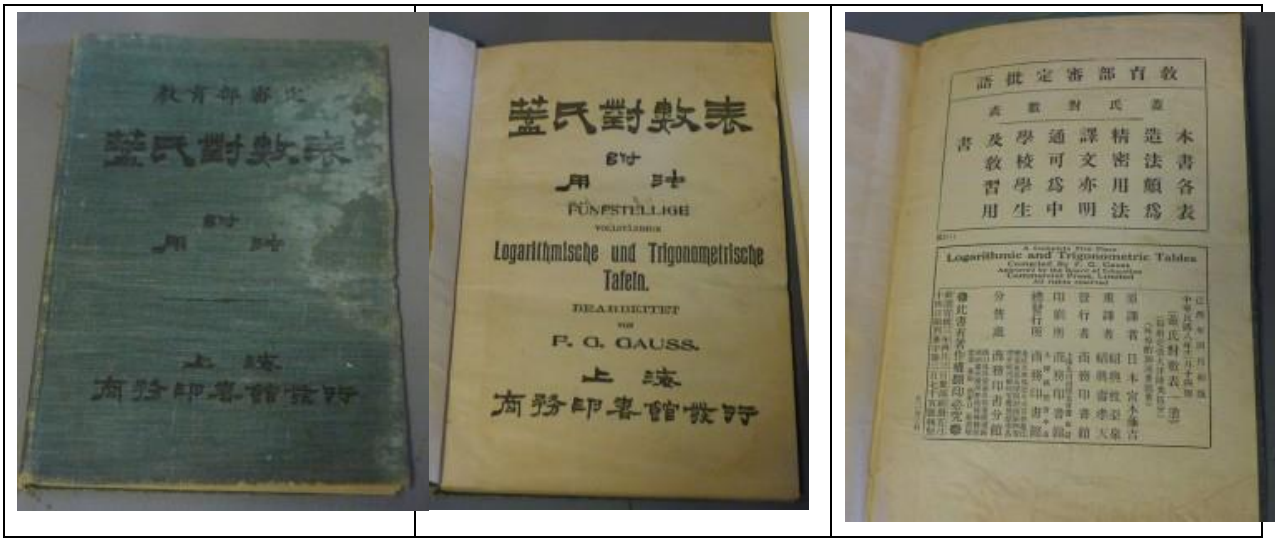


Figure 13: 5 digit Logarithmic and trigonometric tables - title

[19] Mei Juecheng and others, editor. 御製數理精蘊 (Yuzhi Shuli Jingyun).臺北臺灣商務印書館, 1986. [3 volumes, volumes 799–801 of 文淵閣四庫全書];
Denis Roegel: http://locomat.loria.fr/shulijingyun1723/shuli1723intro.pdf

[20] Rainer Heer: http://www.rechnerlexikon.de/artikel/Friedrich_Gustav_Gauss

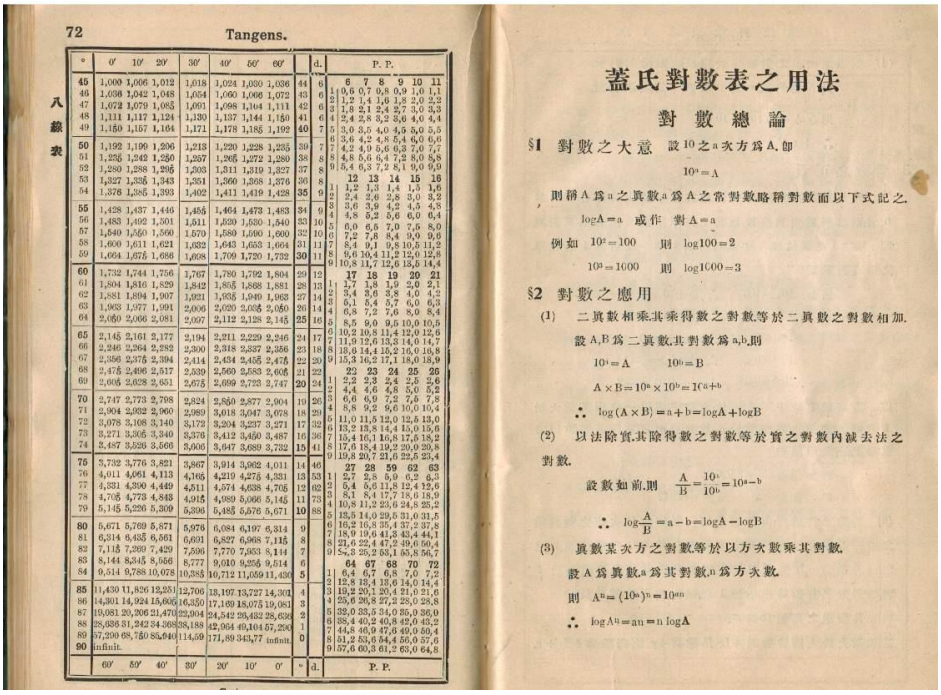
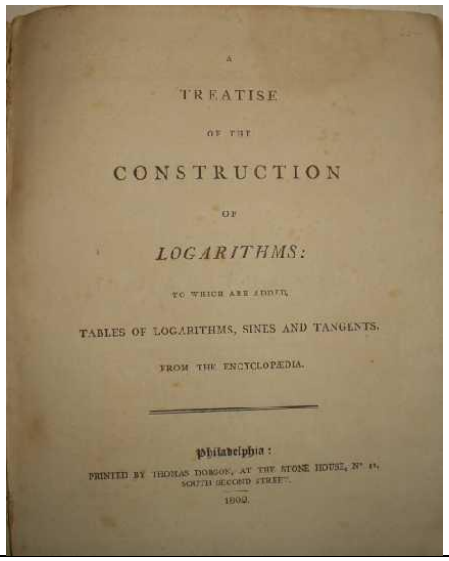


Figure 14: 5 digit Logarithmic and trigonometric tables – start of instructions

The land surveyor *Friedrich Gustav Gauss* from Bielefeld, Germany was not a relative of the famous mathematician *Carl Friedrich Gauss* (1777 – 1855).

5 From Europe to the West – America

In 1802 *Thomas Dobson* (born 1751 near Edinburgh, Scotland - 1823 in Philadelphia, Pennsylvania) published in Philadelphia the *Treatise of the construction of logarithms to which are added, tables of logarithms (to 6 decimals), sines and tangents* as part of the Encyclopedia Britannica. As the title says the first part covers some historical aspects followed by the logarithmic tables and 6 pages with tables of meridonial parts.



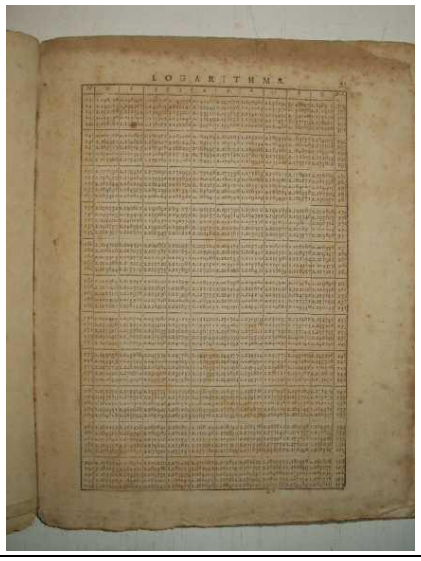


Figure 15: Treatise of the construction of logarithms to which are added, tables of logarithms (to 6 decimals), sines and tangents – title and logarithms

More than 100 years later there was another transfer of logarithms from Europe to the US. The version of *Jean Peters´* (1869 – 1941) Tables of Logarithms to 10 decimals - calculated with a difference calculating machine constructed especially for that reason by Mr. *Christel Hamann* (1870 – 1948) - were copied for the US market. The reason for that copying was, that this table was considered as the most accurate of that time.The Frederik Ungar Publishing Company, New York, did print and publish that copy in the original German language ! in 1957. For more information on some other European originators of US tables see [21].

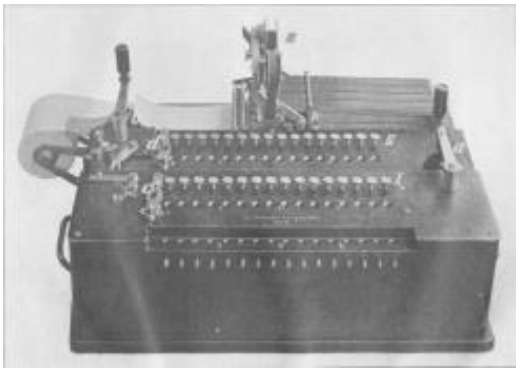


Figure 16: The Hamann Difference Engine. There is no model of the machine nor is the original known to exist any more.[22]

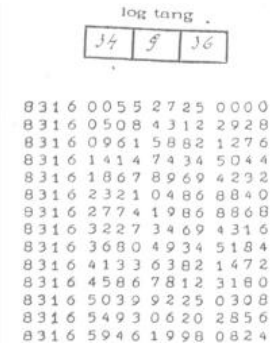


Figure 17: A print out of table-values calculated by the Hamann Difference Engine

[21] Klaus Kühn: German roots for some U.S. tables of logarithms; Proceedings of the IM 2011, (9 -26) Boston; Oughtred Society

[22] Stephan Weiss: http://www.mechrech.info/publikat/HamDiffM.pdf

6. From Europe to the other Continents

*E. W. Hobson´*s (1856 – 1933) article [23] about *John Napier* and the invention of Logarithms from 1914 was eventually spread to several cities as can be seen from the impressum where the Cambridge University Press had affiliations to – New York, Bombay and Calcutta, Toronto, Tokyo . The first paperback edition from 2011 has added some other cities - probably not relevant for 1914.
Alex Craik [24] is wondering why the prominent English mathematician *Hobson* did not contribute to the Proceedings of the Tercentenary Congress in Edinburgh although he was listed as a 'founder member'. Other 'founder members' even came from South Africa, Cairo (Egypt), Lahore (India), Wellington (New Zealand), Sydney (Australia). Their presence indicated the spread of logarithms to those continents. This may indicate the interest in those cities/continents in logarithms.

Robert Shortrede (1800 – 1868), a surveyor, might have been the one who has introduced logarithms to India. He was Captain H.E.I.C.S in the Indian Army and was first assistant of the great trigonometrical survey of India. Later he held the rank of Major General in the Bombay Army.

In 1844 he published his first Logarithmic Tables ("Logarithmic Tables to Seven Places of Decimals Containing Logarithms to Numbers from 1 to 120.000 , Numbers to Logarithms from .0 to 1.00000, Logarithmic Sines and Tangents to Every Second of the Circle, with Arguments in Space and Time and New Astronomical Geodesical Tables" (Tables 1 - 3 edited by *Edward Sang (1805 – 1890)* and 4 - 41 superintended by *William Galbraith (1786 – 1850)*)) in one volume in Edinburgh (Adam Black..; John Murray..; W.H. Allen&Co, bookseller to the Hon. The East India Company). In 1849 Shortrede published ("Logarithmic Tables Containing Logarithms to Numbers from 1 to 120.000 , Numbers to Logarithms from .0 to 1.00000, to Seven Places of Decimals. Tables with Centesimal, and Decimal Arguments for finding Logarithms and Antilogarithms as far as Sixteen and twenty-five places; Tables to five places for finding the Logarithms of the Sums and Differences of Antilogarithms; Tables of Barometric and Thermometricc Heights; together with Several other Tables of frequent use" A second edition appeared in 1858 bound together with the 1849 volume, and a revised version was presented by Major-General *John-Caufield Hannyngton (1807 – 1886)* – a colleague of Shortrede and designer of his famous slide rule - in 1873.

Up to now there is no clear evidence when and where tables of logarithms arrived in Africa, in New Zealand, or in Australia. But there is no doubt that logarithms have been used on those continents too. The main source for logarithms in those continents may have been books of British origin. And were introduced by the first (British) university professors there; but perhaps used before then by colonial administrators and surveryors.

For example one of the main table of logarithms used in Australia was that of *Frank Castle* (1856 – 1928) mainly published by the Macmillan Company, London. The first edition of his "Five-figure logarithmic and other tables" was published in 1909 by Nelson Thornes Ltd.

[23] *Ernest William Hobson: John Napier* and the invention of Logarithms; Cambridge University Press, 1914

[24] Private communication

7. A Documentation of Tables of Logarithms [25]

It seemed fitting to me to mark this milestone of the first Table of Logarithms by chronicling everything known about this revolutionary invention as a compendium - Collectanea de Logarithmis.

The bilingual German/English content of the Collectanea consists of more than 3200 entries representing umpteen articles, many tables and Internet links relevant to the logarithm. This logarithmic "labour of love" took more than 10 years of research to collect and compile.

For example, the topic "Tables of Logarithms" includes a unique 121-page worldwide inventory of more than 3100 tables by the year they were first published, the author, the title, the publisher/city, the language and for many, a link to an online source. However, nearly a quarter of the links is to 700 digitised original tables and most are downloadable. This in turn creates the opportunity to study such tables in-depth and to learn first-hand how their content and design evolved over time.

Whereas the many specialised articles included with the Collectanea compendium cover logarithm-related areas such as science, engineering, economics, transportation and academic study. They provide a theoretical and in-depth insight into the part played by logarithms during the last five centuries. Over 90 of these articles, 27 in English, have never been previously published or are not readily available anywhere else.

So the Collectanea de Logarithmis provides a rich historical account of the role and calculating importance of logarithms. It represents an invaluable resource for further study or reference. Importantly being a unique "one-stop shop", it will doubtlessly save hours of unfruitful Internet surfing and library visits as everything you are ever likely to need is to hand with the Collectanea.

The sheer volume of the contents practically and financially precluded publishing the Collectanea de Logarithmis as a book. So it is only available digitally as a DVD. To protect against plagiarism, copies of the Collectanea come with a USB-stick security device. This PC device activates an offline "home page" on the DVD. Very much mirroring the experience of commercial Internet browsers, the intuitive inbuilt browser makes it easy to navigate through the Collectanea and discover its "Aladdin's Cave" encyclopedia of information.

p.s.: Some figures are examples of the design of tables. If there is interest in more detailed figures, please contact the author.

[25] For more details see www.collectanea.eu