A Joint Meeting of the James Clerk Maxwell Foundation and the British Society for the History of Mathematics in celebration of the 400th Anniversary of the publication of John Napier's *Mirifici Logarithmorum Canonis Descriptio.*

This meeting was held on 4th April in the agreeable surroundings of Clerk Maxwell House, 14 India Street, Edinburgh. The building is an elegant Georgian town house in the district known as New Town and was the actual birthplace of James Clerk Maxwell. The Foundation purchased the house in 1993 and, largely through the energies of Professor David Ritchie, the Foundation Director, the internal rooms of the building were restored close to their original state. Today, many Maxwell memorabilia are present either as portraits on the walls or as objects in display cabinets. The venue was thus somehow infused with an aura of 'great ideas', proving a visually stimulating and most suitable place to hold this celebration of John Napier.

The meeting was extremely well supported and, as space limited attendance to only 40 persons, several people were sadly disappointed - but happily any northern-based BSHM member who wished to attend did so.

We were privileged to begin the day with opening remarks by Sir Michael Atiyah. Sir Michael reflected on the venue and on both Maxwell and Napier. The contribution made by these two great minds to the furtherance of science in general and mathematics in particular was just immense. He was looking forward to the day hugely.

Our first speaker was Prof. Dr. Joachim Fischer who outlined his research into the methods that Napier used to calculate his logarithms to such accuracy. Undoubtedly a difficulty was that Napier's construction of the necessary 'matched' AP and GP became outdated shortly after publication of the *Descriptio*. In Napier's concept his logarithms were designed for division of Sine functions. The values decreased with increasing angle and the logarithm of 1 was not zero. Later Napier's son published his father's *Constructio* which explained in a somewhat 'Euclidian style' what Napier had done but, even so, matters were far from obvious to modern eyes. And in the event, following the famous discussion between Napier and Briggs, the basis for logarithms changed to that with which we are familiar today. Nevertheless it is highly instructive to try to recreate the mathematics that Napier might have used. In doing so, it is clear that Napier was extraordinarily innovative for the era. Besides his first use of the decimal point in print, he had a sound concept of 'difference' calculations and forward prediction of accuracy. Prof. Fischer has summarised his research in a paper in English that he expects to publish shortly.

Dr. Klaus Kuehn then took the floor and introduced himself, amongst other things, as a 'collector' of Logarithm Tables. Dedicating his talk to the late Thomas Wyman, Emeritus President of the Oughtred Society, who had died in March 2014, he then both

enthralled and entertained us with the astonishing story of how Logarithm Tables promulgated throughout Europe and beyond. By looking at the actual figures together with any known errors, dependences on earlier works can be established. A time diagram of numbers of different books of tables published revealed a value of 'five per year' between 1614 and 1814, then peaking to about '20 per year' until 1983 and the advent of the calculator. Books had been published in about 170 cities worldwide. Taking Russia as a particular area, he explained that Peter the Great in 1700 had founded a school of Mathematics and Navigation, inviting a Scot, Henry Fargwarson, to direct it. It is highly probable that Fargwarson introduced Vlacq's 1670 tables to Russia at this time. He outlined similar transmissions to China by the Jesuits and to the USA in 1802. Tables had also been published in Arabic in Istanbul – a hint as to why this might have happened was revealed by another speaker in a later talk. Altogether a very intriguing exposition.

In 1617, just prior to his death, Napier's book *Rabdologiae* was published. In this book Napier described the lattice calculating 'rods or bones' that bear his name. Professor Michael Williams took us through the use and development of the 'bones'. Effectively the rods were merely a collection of all possible columns of a 'Gelosia style' lattice multiplication table. Others had done similar things, in particular William Stickard, a polymath professor at Tubingen, had invented a calculating machine in 1623 that worked on similar principles. In 1668 Gaspard Schott had made a device with Napier's rods inscribed on cylinders. In 1679 Samuel Moreland had made a similar calculating machine for Cosimo de Medici. Nevertheless there was always the problem of 'carry digits' with these mechanisms. But in 1885 Edouard Lucas and Henri Genaille devised a set of 'bones' with 'chevrons' attached that indicated how and where to insert the 'carries'. Finally the very early 1620 IBM computer, manufactured in 1960, a so called 'CADET' machine, used look up tables based on Napier's 'bones' to do the arithmetic! Would that Napier could have seen it!

In another section of *Rabdologiae* Napier described his so termed 'Chessboard Abacus' and Dr Steve Russ showed us how it worked. Your writer found this device simply astonishing, as did others. It is clearly a 'binary arithmetic' device conceived many years before the generally accepted first description by Leibniz in 1679. The term chessboard comes from the 'square' grid of the table, which is used 'point down' with the margins marked with increasing powers of two. Counters are used on the board and the calculation steps are done by moving the counters directly along the grid like a 'rook', or diagonally like a 'bishop'. It is possible to accomplish all arithmetic operations and extraction of roots. Dr Russ then emphasised the connection between the ideas found in this 'abacus' and modern concepts of 'human computing'.

Dr. Gerlinde Faustmann took us through the history of and personalities involved with the Logarithm tables published in Austrian Empire in the early 1800s. She discussed the roles of Georg Vega, Leopold Unterberger, Ignaz Lindner and Johann Pasquich. Vega worked within a military environment and published several works including logarithmic and trigonometric tables; he was by reputation a prodigious calculator. He sadly died by drowning in the Danube at the early age of 48. Unterberger also had a military background, knew Vega well and published his own tables of logarithms in 1777,

together with many examples of their use. Somewhat later, Lindner, another military officer who had been taught by Vega, published a number of mathematical texts including a set of Logarithms. Similarly Pasquich, a consecrated priest, took up mathematical posts at the Universities of both Pest and Buda. He is perhaps most famous for being accused of falsifying results of a 'comet' prediction – but the outcome was favourable to him as he was completely exonerated and gained fame because of that. His logarithms were published in 1817.

For the final talk of the morning session, Prof. Martin Campbell-Kelly brought to our attention the fascinating history of Charles Babbage's tables of Logarithms published in 1827. These were 'logarithms in colour' in that Babbage specified different coloured papers for their printing. This was to make them easier to read. Babbage had always had great interest in 'tables' generally, producing 'star' tables with Herschel in 1820, actuarial tables in 1824 and then logarithms with collaborators Francis Baily and Thomas Colby. His source would appear to be have been Callet's 1795 edition and he experimented with both typeface and font. Prof. Campbell-Kelly made it abundantly clear with comparison slides that Babbage's readability on yellow paper was greatly superior to, say, Callet in black on white. Subsequently various other editions were published often using a variety of colours - white, fawn, pale yellow and grey. The tables remained extant until the last edition of 1915 but coloured paper was dispensed with from 1841. The audience were agreed that 'pink' was the least successful colour apart from the frankly bizarre gold ink on green and red vellum, the latter two being merely specimens and not from any print run! Inevitably the tables contained some errors; five major ones in the first edition that were found after stereotyping was introduced for later works. This was a fascinating and unexpected insight into Babbage's early efforts with tables and before his famous difference engine designs.

After a very full and pleasing morning session, the meeting enjoyed a most delicious buffet prepared by the Clerk Maxwell House caterers in the elegant and historic environment of the 'Board Room'. This was also a good opportunity for some social and professional exchange.

Returning to the Conference Room, Dr Ulf Hashagen explored the background to the 1910 calculation of 8 figure logarithm tables by Bauschinger and Peters using a 'difference engine'. The motivation for the new calculation was that existing fully accurate 7 figure tables were inadequate for the increased precision of astronomy, in particular for the calculation of the ephemeris of minor planets and sun parallax. The requirements were to produce a faultless table in a suitable format that allowed quick interpolation and clear typographical grouping. As Dr. Hashagen remarked, these latter criteria were conflicting! The trade off between a huge table with many interpolations that ensured accuracy together with the need to minimise cost occupied much thinking time for the project. As ever, the logistics of production were paramount. It was estimated that, if working six hours a day, it would take four years to hand compute the data, two years to do so using a machine, four years to copy the work and a further two years to check the data – an enormous undertaking! Further refinement, including the construction of a special difference engine, reduced the estimated time to five years in

total at a cost of 50,000Marks. Work started in the spring of 1908 and by the end of 1910 two volumes were ready for printing. Clearly the difference engine had made a dramatic difference to computing time and Hashagen went on to describe the maker, Christel Hamann, and features of his machine. Bauschinger and Peters's success and almost incredible speed can be put down to an almost perfect management combination of machinery, existing 12 figure tables and excellent human computers. Babbage's dream of creating a perfect table had been realised.

For the final academic talk of the afternoon, Prof. Alex Craik outlined the involvement in logarithms of three Scottish mathematicians, Cargill Knott, Edward Sang and William Spence. He explained that he would take them in reverse chronological order. Knott, who died in 1922, was prolific and wide-ranging author who wrote mainly on magnetism, seismology and quarternions. He had held many academic posts but latterly had been reader in physics at Edinburgh. He was appointed FRS in 1920. He was the leading organiser of the 300th Napier Anniversary, held in 1914, and had edited the famous 'tercentenary volume' of papers that followed. He had provided an article entitled *Edward Sang and his Logarithm Calculations.* Sang who died in 1890 was a mathematical and computational prodigy who entered Edinburgh University at the age of 13. He subsequently worked as a consulting actuary, surveyor, engineer, mathematics teacher and lecturer. He then became professor of mechanical sciences at Manchester New College, before taking up a post in Constantinople where he established an engineering school, and planned railways and an iron works. But he is best known for his logarithmic and other tables compiled over many years with assistance from his two daughters. In 1859 he published a book of 5 figure logarithms and in 1871 a similar volume to 7 places. A proposed volume of 9 figures remained in manuscript. William Spence of Greenock who died in 1815 attended no university. For some years he lodged with William Struthers, a banker, who probably taught him mathematics and who possessed an extensive library. In 1809 Spence published privately a book on Logarithmic Transcendents, now called polylogarithms. Spence tabulated a function known a Dilogarithm, which is sometimes named after him. His early death precluded publication of much of his work but in 1819 Galt and Herschel compiled a collection of his papers. Herschel spoke very highly of Spence's ability and mourned his early loss. During questions, Klaus Kuehn, who had spoken in the morning, said that he now realised the probable source of the Arabic logarithm tables, printed in Constantinople, that he had collected. It was almost certainly Sang's influence that caused them to be printed – the timing fitted very well. This point provided a neat closure to the day's academic talks.

Dr Tayce Phillipson, of the National Museum of Scotland, then gave us a brief introduction to the 'Power of Ten' exhibition put on by the Museum in celebration of the 400th anniversary. Following this, we boarded the coach and set out for Chambers Street and the museum itself. Although small in comparison with the magnificent Grand Gallery of the museum, the 'Power of Ten' display area proved a perfect situation for a number of Napier related artefacts. Your writer would like to mention but two. The first, a set of Napier's 'Bones' with the 'carry' chevrons – a truly beautiful object; the second a large stuffed black cockerel! It is said that Napier owned such a bird and that his

household believed that it had mystic powers. Seeking a thief amongst his staff, Napier required them all touch the cockerel on the back, telling them that it could detect the culprit but not telling them that he had covered it in soot. The miscreant, too scared to touch the bird, was easily found from his clean hands!

From Chambers Street we boarded the coach again to our final destination, John Napier's home at Merchiston Castle. This splendid 15th century fortified tower house stands prominently in the entrance courtyard of the Napier University campus that surrounds it. We were privileged to be shown round the restored house by Charles and Pat Napier, the former a descendent of John and the latter a University librarian and an expert on the Napier family and John in particular. The rooms and staircases of the castle were narrow and we were required to split into two groups for the visit. Whilst waiting, Napier University extended us some welcome hospitality. To say that the building was 'atmospheric' would be a considerable understatement. Whilst ascending the tight spiral staircases between floors, your writer was reminded of the scene set in the 'House of Shaws' in R L Stevenson's *Kidnapped* – fortunately for us the extensive restoration of Merchiston had ensured that no steps gave way! Pat and Charles gave us a most thorough and spirited account of Napier's life and times. One could almost see the great man sitting in the corner of the Great Hall, managing his extensive estates, writing his religious tracts and, in between, calculating his logarithms – most certainly we could sense him.

With that vision in all of us, we departed happily for our various destinations. It had been a quite splendid day!

Thanks are due to the many people who made it all possible – too many to mention by name. But I would like to single out Dr David Forfar of the JCMF whose vision of what was required to make a suitable commemoration was brought to a fine reality. The event was 'free' and the necessary funding was generously provided by JCMF, BSHM, GMS and the EMS. Several people have asked me whether it is intended to produce a 'volume of papers' similar to that edited by Knott for the tercentenary celebrations in 1914. The short answer is no. But what we do intend to create is an 'on-line' resource comprising of summaries of the talks given and other relevant Napier documents. Dr. Klaus Kuehn has kindly agreed to act as a focus for this but any BSHM inquiries may be made to me.

Richard Simpson, Hon Sec, BSHM