Mirifici logarithmorum canonis constructi; et eorum ad naturales ipsorum numeros habitudines; una cum appendice, de aliâ eâque præstantiore logarithmorum specie contenda. Quibus accessere propositiones ad triangla sphærica faciliore calculo resolvenda: Unà cum annotationibus aliquoot doctissimi D. Henrici Briggii, in eas & memoratam appendicem.

Year: 1619
Place: Edinburgh
Publisher: Andrew Hart
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Pagination: 67, [1]
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Reference:

Henderson, James; Bibliotheca Tabularum Mathematicarum. Being a descriptive catalogue of Mathematical tables. Part I, Logarithmic tables (A. Logarithms of numbers), Cambridge, Cambridge University Press, 1926, #6.0, p. 29;

Glaisher, James Whitbread Lee; et al.; Report of the Committee on Mathematical Tables, London, Taylor & Francis, 1873, p. 156;

Horblit, Harrison D.; Collector's Choice: A selection of books and manuscripts given by Harrison D. Horblit to the Harvard College Library, The Houghton Library, Cambridge, MA, 1983, #37, p. 33;

Horblit, Harrison D.; One Hundred Books Famous in Science, New York, Grolier, 1964, #77b

Notes on John Napier and the book

John Napier was born into a leading, prominent family of Scottish lairds (wealthy landowners). The family surname is seen in early documents as Napeir, Nepair, Nepeir, Neper, Napare, Naper, Naipper and the present-day Napier. Little is known about John Napier's childhood and youth. He enrolled at St. Andrews University at the age of thirteen, but there is no record that he ever graduated. Napier later wrote that his fervent interest in theology was kindled at St. Andrews. It is probable that he left St. Andrews to study in Europe, and it must have been there that he acquired his knowledge of higher mathematics and his taste for classical literature.

In 1572, just about the time of his marriage, Napier received title to the family estates. When time permitted from the daily running of his estates, John Napier played an active role in the Scottish Protestant reform movement. What time he had left he used to study mathematics. He is best known today for his invention of logarithms, but in his own time he was best known for his religious commentaries.

Napier's first book on logarithms was one of the most influential mathematical books ever published. It introduced the world to the concept of logarithms and their use. By simplifying arduous calculation, that is, by reducing multiplication and division to addition and subtraction, logarithms became the fundamental principle behind most of the methods of, and aides to, computation prior to the invention of the electronic computer. They also proved to be a fundamental component of many mathematical systems.

After Napier had published the description (see Napier, John; *Mirifici logarithmorum canonis descriptio*, 1614) and the table of his logarithms, his intention was to publish a book describing how they had been calculated. He died before he could complete the task, but his son Robert Napier completed and published it in 1619. Napier's 1614 publication is always referred to as the *Descriptio*, and the 1619 volume as the *Constructio.*

While the *Descriptio* was reprinted many times, the *Constructio*, lacking any tables of logarithms, was of interest only to mathematicians and table makers and thus had far less attention paid to it. The *Descriptio* was translated into other languages almost as soon as it appeared, while the *Constructio* had to wait until 1889 before an English version was produced (see Napier, John [William Rae Macdonald, translator]; *The construction of the wonderful canon of logarithms…*, 1889). The notes on individual pages presented here are based largely on the English translation by Macdonald.

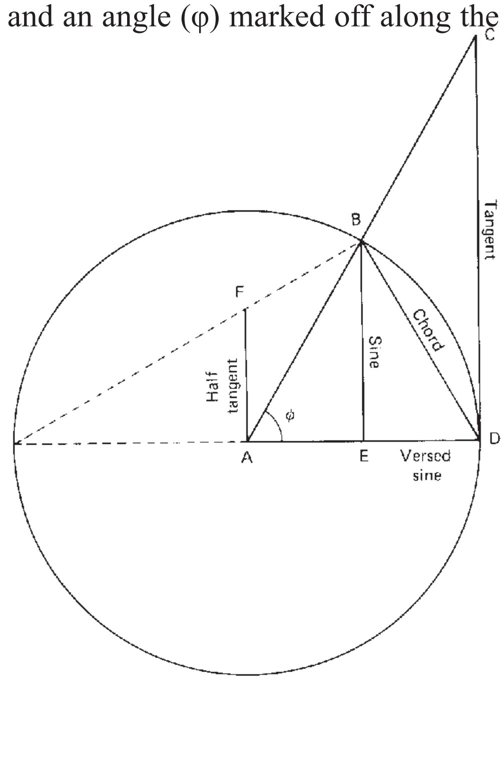
A detailed description of Napier's methods of calculating logarithms can be found in the paper: Carslaw, H. S., "The discovery of logarithms by Napier," *Mathematical Gazette*, Vol. VIII, 1915-1916, pp. 76-84, 115-119.

This work was issued in a confusing manner. It contains a collective title page very similar to that of the *Descriptio* (but without any *Descriptio* text) followed by the title page of the *Constructio.*

Notes on the old forms of trigonometric functions

At the time of this publication, trigonometric values (chords, secants, sines, versed sines (cosines), tangents, half tangents and so on) were not usually defined as they are today (in which the functions, such as the sine, are ratios of the length of two sides of a triangle).

The above figure shows a circle and an angle (φ) marked off along the circumference. With respect to the



given radius, the various trigonometric functions were defined as the lengths of specific lines, for example:

- chord of φ was the length of the line BD,
- sine of φ was the length of the line BE,
- versed sine of φ was the length of the line DE,
- tangent of φ was the length of the line DC,
- half tangent of φ was the length of the line AF (the half tangent is really the tangent of half the angle), and
- secant of φ was the length of the line AC

General notes on the condition of older books

Books as old as this usually suffer from some problems just because of the wear they have been subjected to over the many years of their existence. One usually noticeable condition item is known as *browning* or *foxing* of the paper - usually brown or yellow areas due to the chemical action of a micro-organism on the paper. This can vary dramatically from page to page, often depending on such variables as the contents of the paper used, the composition of the ink used by the printer, and the dampness (or lack of) that the work has been exposed to over the years. Where these images were badly foxed, some slight manipulation of the intensity of the colors has been done to ease the reading of the foxed page. Any other notable condition problem will be commented upon near the image concerned.

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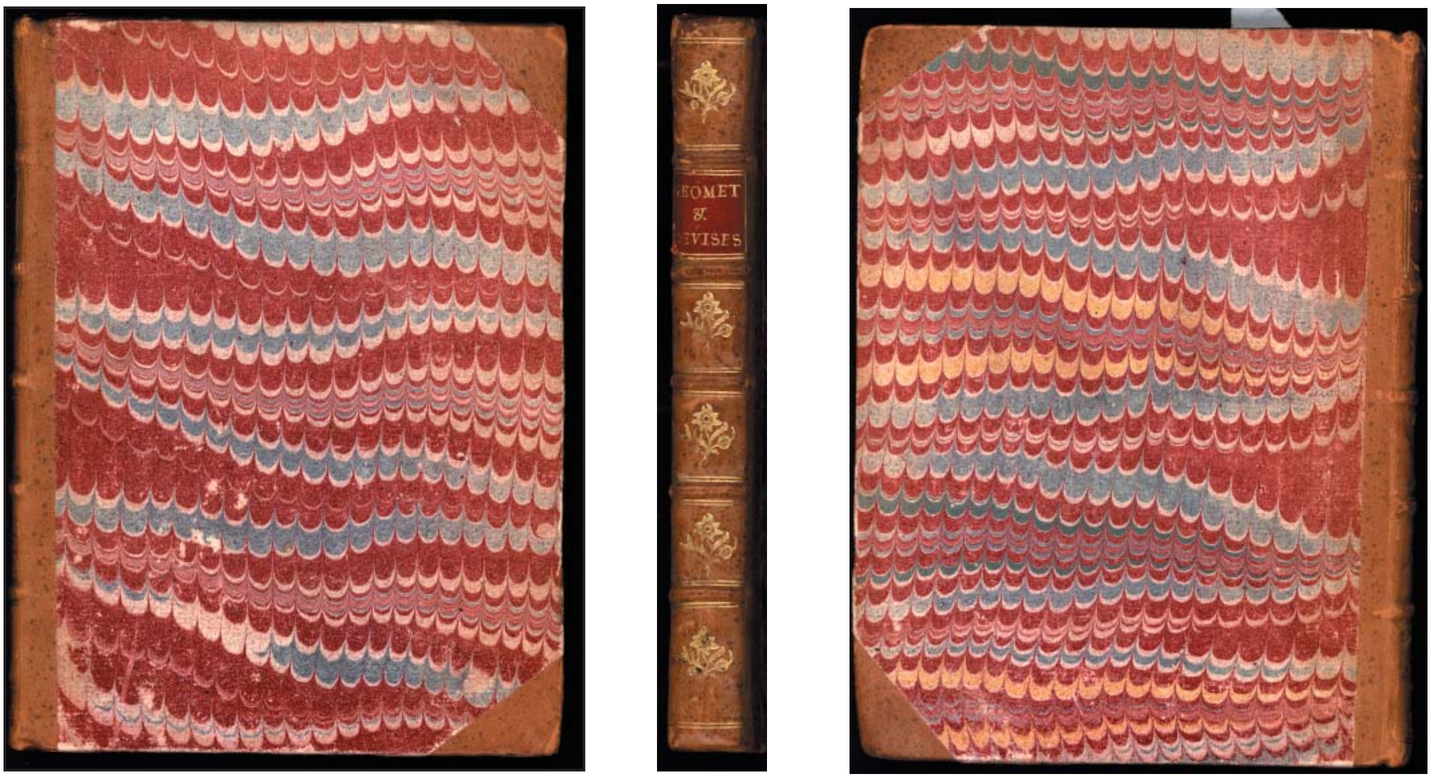
From the Tomash Library on the History of Computing

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



The front cover, spine and rear cover of this volume. The binding dates from the 18th century.

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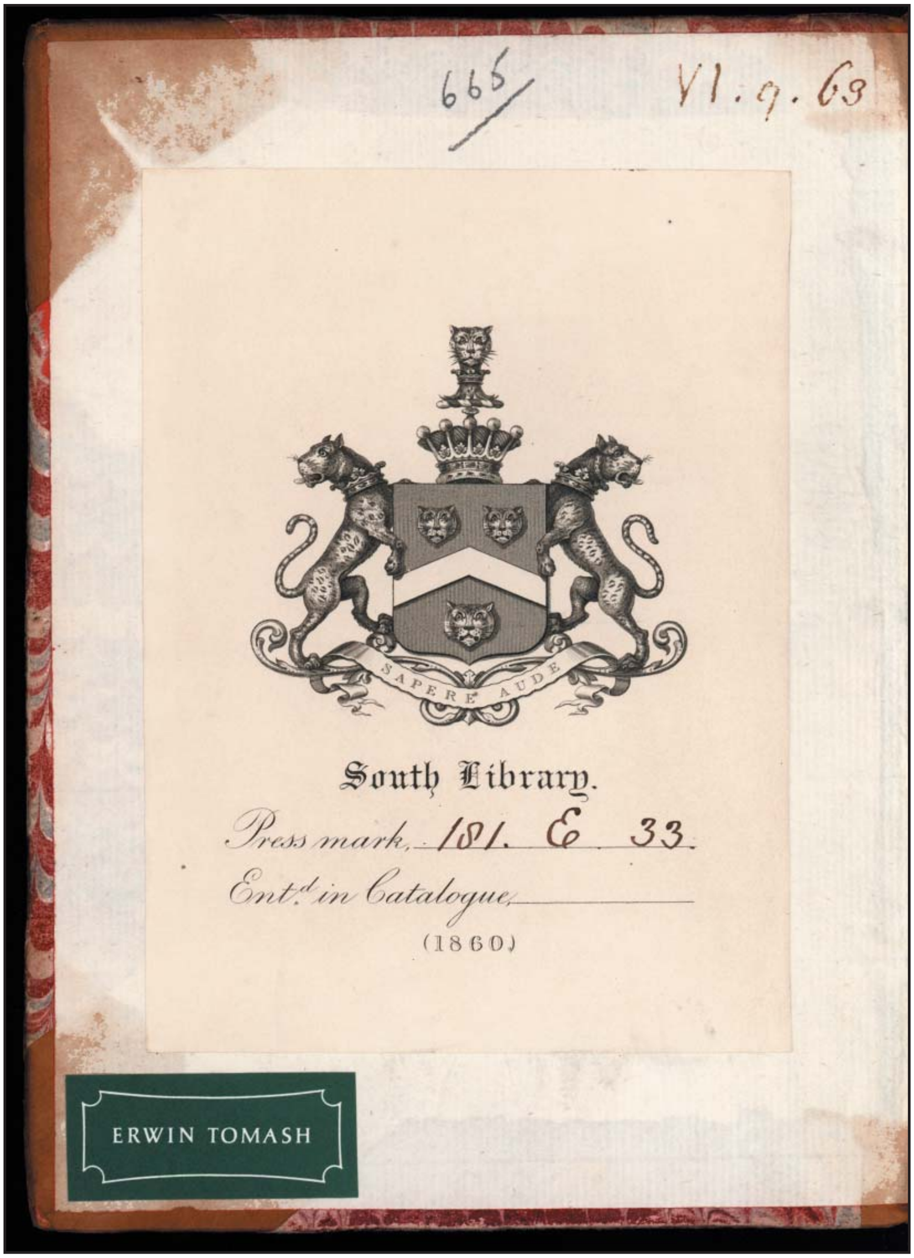
Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



This book was purchased for the Tomash Library from the fourth Sotheby's sale of the Macclesfield library in November of 2004. The large bookplate denotes the Macclesfield South Library and their shelf mark *181. E. 33*.

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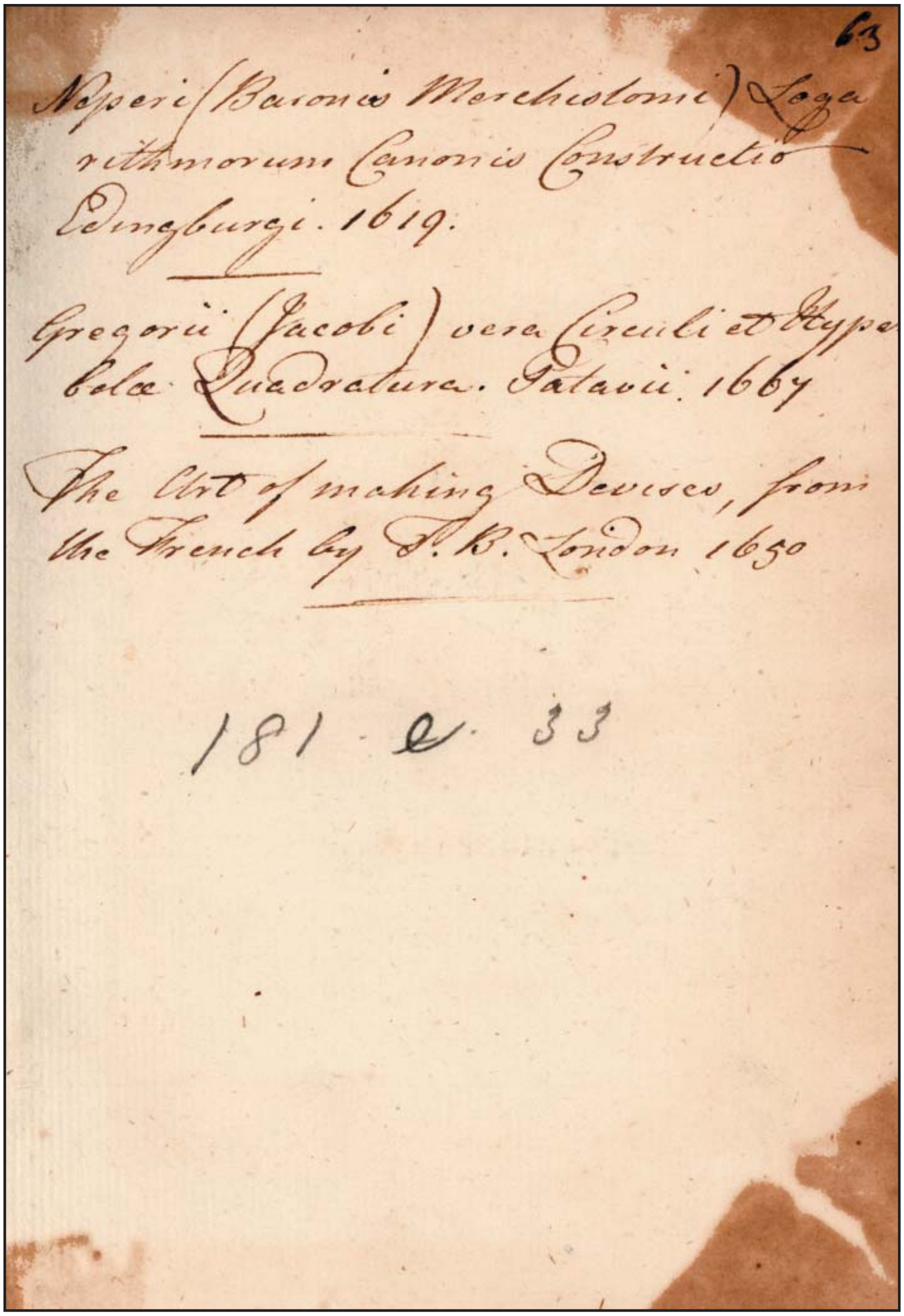


63

Naperi (Baronis Merchistonii) Logarithmorum Canonis Constructio Edingburgi. 1619.

Gregorii (Jacobi) vera Circuli et Hyperbolae Quadratura. Patavii. 1667

The Art of making Devises, from the French by T. B. London 1650

This volume is a sammelband (the binding together of different works into one volume) of three works that are listed on the recto of the free endpaper. Only the work by Napier (*Constructio*) in included in this file.

The verso of the free endpaper.

From the Tomash Library on the History of Computing

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh

MIRIFICI

LOGARITHMORUM CANONIS DESCRIPTIO,

Ejusque usus, in utraque Trigonometria; *ut etiam in omni Logistica Mathematica*, amplissimi, facillimi, & *expeditissimi explicatio.*

ACCESSERVNT OPERA POSTHVMA;

Primò, Mirifici ipsius canonis constructio, & Logarithmorum ad naturales ipsorum numeros habitudines.
Secundò, Appendix de alia, eáque præstantiore Logarithmorum specie construenda.
Tertiò, Propositiones quædam eminentissimæ, ad Triangula sphærica mirâ facilitate resolvenda.

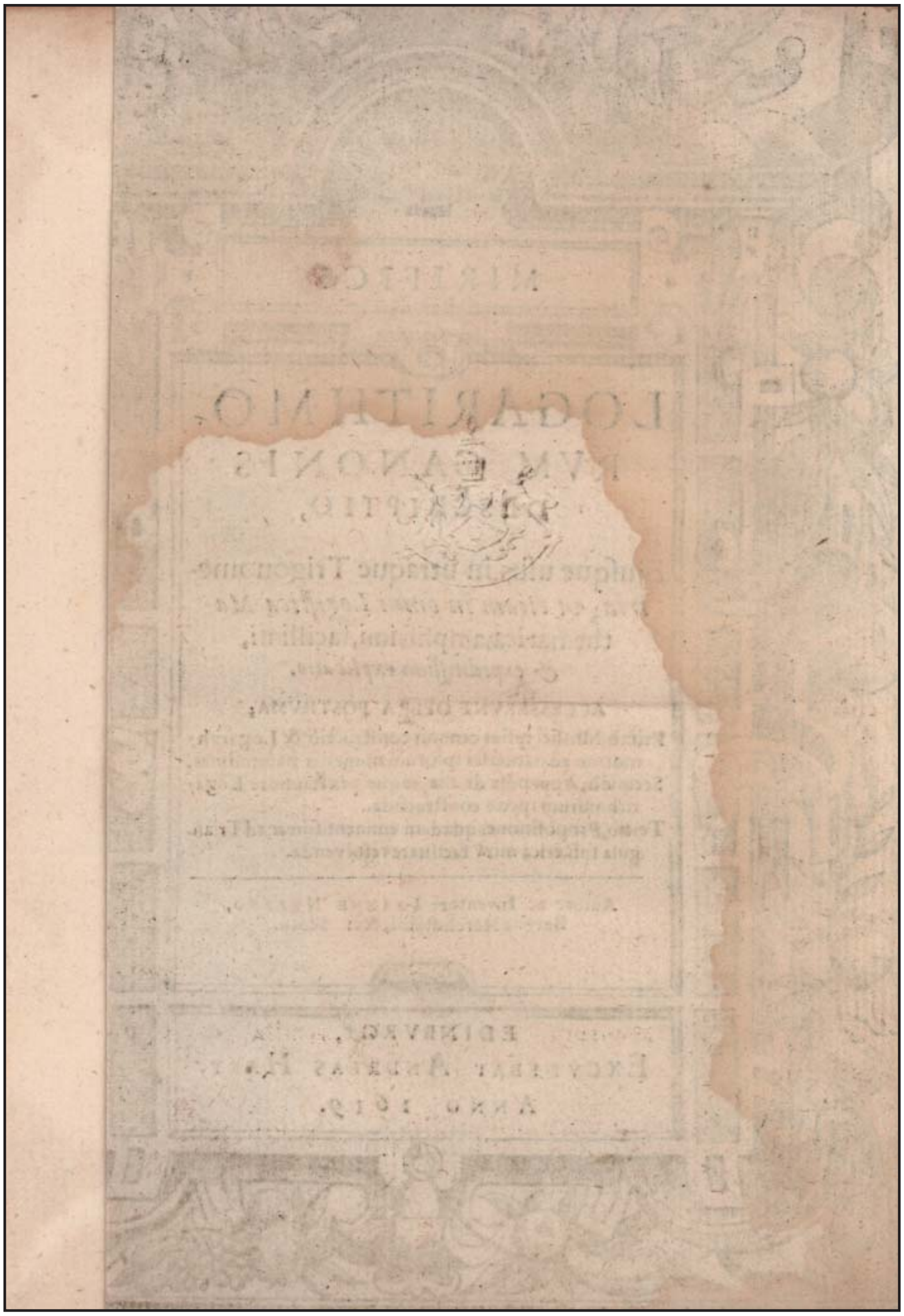
Autore ac Inventore IOANNE NEPERO, Barone Merchistonii, &c. Scoto.

EDINBVRGI,
EXCVDEBAT ANDREAS HART.
ANNO 1619.

While this title page appears to be that of the *Descriptio*, it is actually the collective title page mentioned in the introductory notes.

From the Tomash Library on the History of Computing

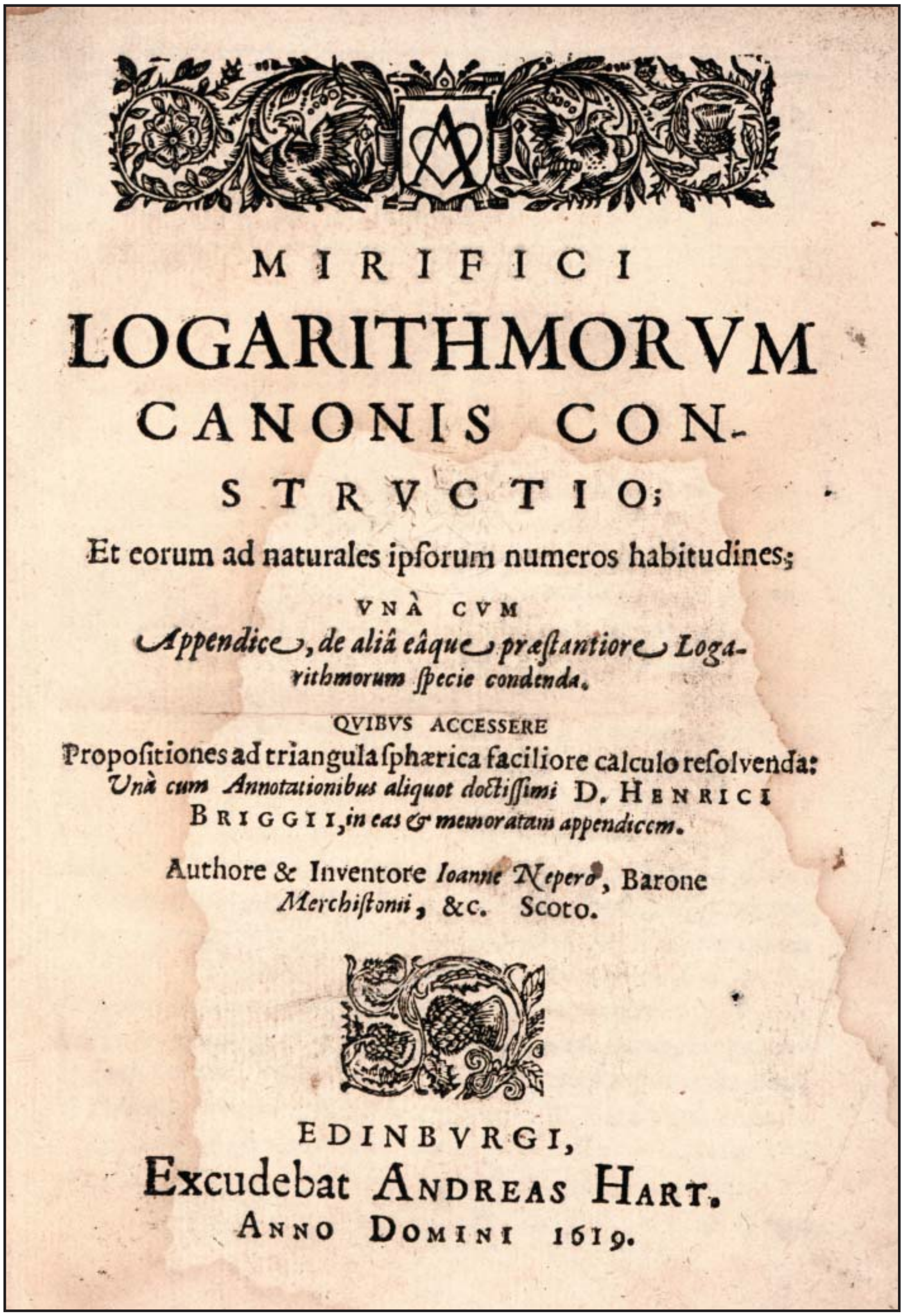
Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



The verso of the collected title page.

From the Tomash Library on the History of Computing

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



MIRIFICI
LOGARITHMORVM
CANONIS CON-
STRVCTIO;
Et eorum ad naturales ipsorum numeros habitudines;
VNÀ CVM
Appendice, de aliâ eâque præstantiore Logarithmorum specie condenda.
QVIBVS ACCESSERE
Propositiones ad triangula sphærica faciliore calculo resolvenda:
Unà cum Annotationibus aliquot doctissimi D. HENRICI BRIGGII, *in eas & memoratam appendicem.*
Authore & Inventore *Ioanne Nepero*, Barone *Merchistonii*, &c. Scoto.

EDINBVRGI,
Excudebat ANDREAS HART.
ANNO DOMINI 1619.

The title page of this volume: Construction of the wonderful table of logarithms; and their relation to their natural numbers; with an appendix on the making of another and better type of logarithm. In addition to which are propositions for solving spherical triangles. Together with notes by Henry Briggs. By the author and inventor John Napier, Baron of Merchiston, etc. a Scot. Printed by Andrew Hart, Edinburgh, 1619.

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Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh

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Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh

LECTORI MATHESEOS STVDIOSO

S.

ANTE *aliquot annos (Lector Philomathes) Mirifici Logarithmorum Canonis vſum, memoriæ ſemper colendæ parens publici Iuris fecerat; ejus verò ſyntaxin ac creandi methodum, vt ipſe monuit Pag. 7ª. & vltimâ Logarithmorum, certo conſilio Typis committere noluit; donec quodnam eſſet eorum, qui in hoc doctrinæ genere verſati ſunt, de hoc Canone Iudicium ac cenſura exploratum habuiſſet. Mihi verò, poſt ipſius ex hâc vitâ commigrationem certis tecmeriis conſtat, Methematicarum diſciplinarum peritiſsimos novum hoc Inventum plurimi facere; & nihil iis gratius accidere poſſe, quàm ſi Mirifici hujus Canonis conſtructio, aut ea ſaltem, quæ ipſi aliquid lucis afferre poſsint, publicæ vtilitatis gratiâ in lucem prodeant. Quamvis igitur mihi probè perſpectum ſit, ipſum authorem huic opuſculo extremam manum non impoſuiſſe; feci tamen quantum in me fuit, vt horum honeſtiſsimo deſiderio ſatisfieret, eorumque ſtudiis præſertim qui imbecilliores ſunt, & in ipſo limine hærere ſolent, hac in parte conſuleretur. Nec dubito, quin hoc opus poſthumum multò perfectius ac elimatius in lucem prodiiſſet, ſi ipſi authori patri chariſsimo (in quo, ex optimorum hominum ſententiâ, inter alia præclara hoc eximii eminebat, res difficillimas methodo certâ &*

 facili

Robert Napier writes a preface. It reminds readers that his Father had published his book on logarithms several years earlier and, in it, he mentioned that he would write another explaining how they were calculated if the mathematicians thought them worthy of his efforts. Robert indicated that his father died before finishing this book and he has taken up the task of completing and publishing it. It seemed reasonable to add to Napier's work an appendix explaining a new kind of logarithm that he had mentioned in the introduction to his book Rabdologiae (the logarithms that he and Briggs had agreed were better - those to base 10). He has also included an appendix by Henry Briggs that comments on the new logarithms and the rules Napier had set down for solving trigonometric problems in spherical triangles.

facili, quàm paucissimis expedire) Deus longiorem vitæ vsuram concessisset. Habes igitur (Lector benevole) in hoc libello, doctrinam constructionis Logarithmorum (quos hîc numeros artificiales appellat; hunc enim tractatū, ante inventam Logarithmorum *vocem, apud se per aliquot annos conscriptum habuerat) copiosissimè explicatam; in qua eorum natura, symptomata, ac variæ ad naturales eorum numeros habitudines perspicuè demonstrantur. Visum est etiam ipsi syntaxi subnectere Appendicem quandam, de alia Logarithmorum specie multò præstantiore condenda, (cujus, ipse Iuventor in Epistola* Rabdologiæ *suæ præfixa meminit) & in qua Logarithmus vnitatis est* 0. *Hanc loco vltimo vltimus ejus labor excipit, ad vlteriorem Trigonometriæ suæ Logarithmicæ perfectionem spectans; nempe propositiones quædam eminentissimæ, in Triangulis sphæricis non quadrantalibus resolvendis, absque eorum in quadrantalia aut rectangula divisione, & absque casuum observatione: quas quidem Propositiones in ordinem redigere, & ordine demonstrare statuerat, nisi nobis morte præproperâ præreptus fuisset. Lucubrationes etiam aliquot, Mathematici excellentissimi* D. Henrici Briggii *publici apud* Londinenses *Professoris, in memoratas Propositiones, & novam hanc Logarithmorum speciem, Typis mandari curavimus; qui novi hujus Canonis supputandi laborem gravissimum, pro singulari amicitiâ quæ illi cum Patre meo* L. M. *intercessit, animo libentissimo in se suscepit; creandi methodo, & vsuum explanatione Inventori relictis. Nunc autem ipso ex hâc vitâ evocato, totius negotii onus doctissimi* Briggii *humeris incumbere, & Sparta hæc ornanda illi sorte quadam obtigisse videtur. Hisce interim (Lector) laboribus quibuscunque fruere, & pro humanitate tuâ boni consulito.* Vale.

ROBERTVS NEPERVS, F.

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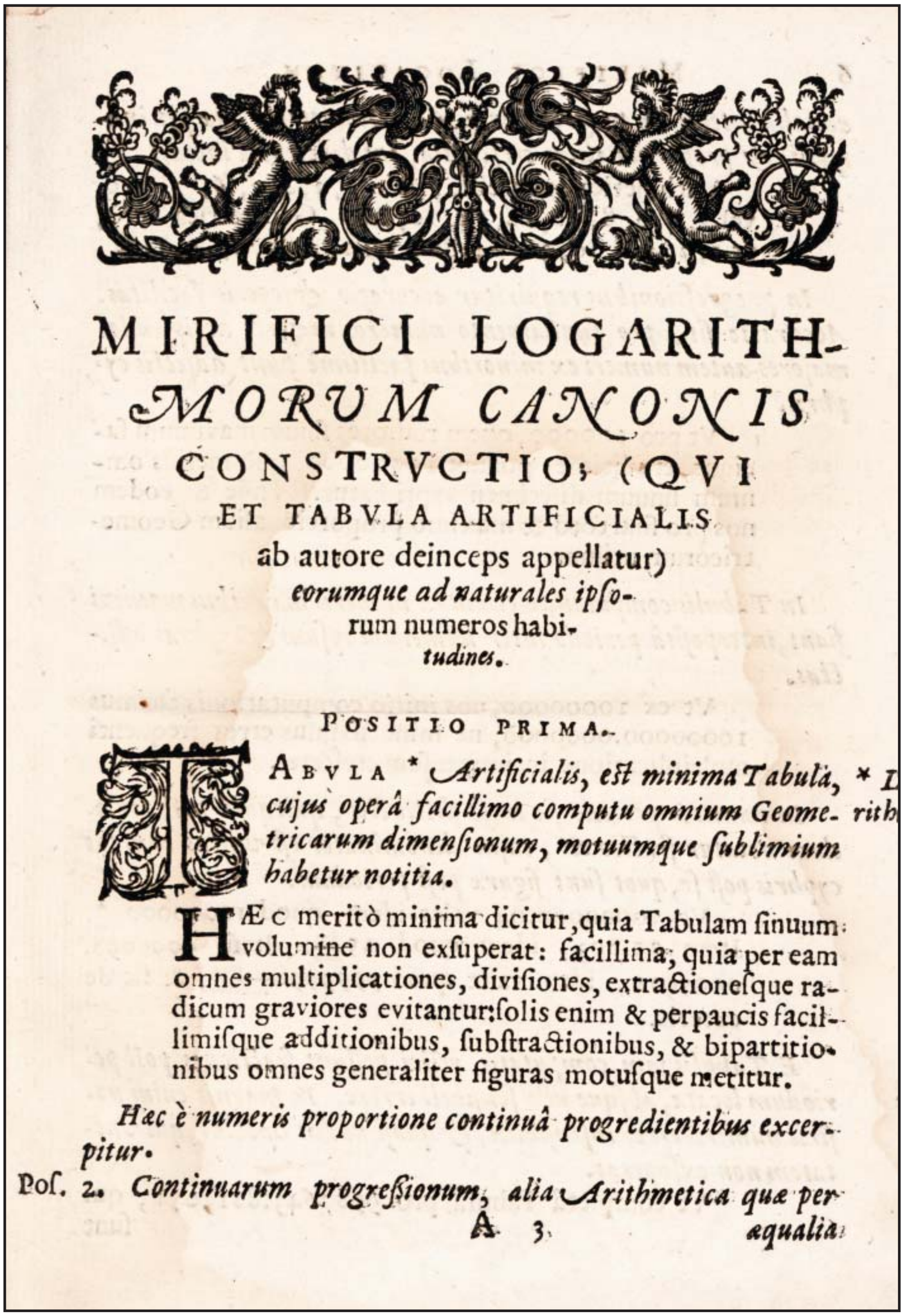
Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



MIRIFICI LOGARITHMORUM CANONIS CONSTRUCTIO; (QUI ET TABULA ARTIFICIALIS ab autore deinceps appellatur) *eorumque ad naturales ipsorum numeros habitudines.*

POSITIO PRIMA.

TABULA * *Artificialis, est minima Tabula, cujus operâ facillimo computu omnium Geometricarum dimensionum, motuumque sublimium habetur notitia.*

HÆc meritò minima dicitur, quia Tabulam sinuum volumine non exsuperat: facillima, quia per eam omnes multiplicationes, divisiones, extractionesque radicum graviores evitantur; solis enim & perpaucis facillimisque additionibus, substractionibus, & bipartitionibus omnes generaliter figuras motusque metitur.

Hæc è numeris proportione continuâ progredientibus excerpitur.

Pos. 2. *Continuarum progressionum, alia Arithmetica quæ per æqualia*

A 3.

Napier’s work consists of 59 propositions which describe everything from a logarithm table to its final construction.

Proposition 1: A table of logarithms (Napier refers it as a table of “artificial numbers” as this was the term he first used to describe them) is small, but with it one can do multiplication, division, and extraction of roots.

æqualia intervalla progreditur: Alia Geometrica, quæ per inæqualia & proportionaliter crescentia, aut deficientia incedit.

Arithmetica progressio, ut 1, 2, 3, 4, 5, 6, 7, &c. vel 2, 4, 6, 8, 10, 12, 14, 16, &c. Geometrica verò, ut 1, 2, 4, 8, 16, 32, 64, &c. vel 243, 81, 27, 9, 3, 1.

3. *In progressionibus requiritur accuratio, & operis facilitas. Accuratio fit, pro fundamento numeros majores accipiendo: majores autem numeri ex minoribus facillimè fiunt adjectis cyphris.*

Vt pro 100000, quem rudiores sinum maximum faciunt, eruditiores ponunt 10000000, quò meliùs omnium sinuum discrimen exprimatur. Vnde & eodem nos pro sinu toto & maximo proportionalium Geometricorum utimur.

4. *In Tabulis computandis etiam ex numeris majoribus maximi fiant, interpositâ periodo inter numerum ipsum & cyphras adjectas.*

Vt ex 10000000, nos initio computationis facimus 10000000.0000000, ne minutissimus error frequenti multiplicatione in immensum cresceret.

5. *In numeris periodo sic in se distinctis, quicquid post periodum notatur fractio est, cujus denominator est unitas cum tot cyphris post se, quot sunt figuræ post periodum.*

Vt 10000000.04 valet idem, quod $10000000\frac{4}{100}$. Item 25.803, idem quod $25\frac{803}{1000}$. Item 9999998.0005021, idem valet quod $9999998\frac{5021}{10000000}$, & sic de cæteris.

6. *E Tabulis jam computatis, rejici possunt fractiones post periodum locatæ, absque ullo sensibili errore. In magnis enim nostris numeris error insensibilis, & quasi nullus habetur, qui unitatem non exsuperat.*

Vt completâ Tabulâ pro 9987643.8213051, qui

sunt

Proposition 2: Progressions of numbers of are two types: arithmetic and geometric. Arithmetic progressions are ones with equal intervals between entries (e.g., 1, 2, 3... or 2, 4, 6,...) while geometric progressions are those with increasing or decreasing intervals (a constant being used as a multiplier) (e.g., 1,2,4,8,... or 243, 81, 27, 9, 3, 1).

Propositions 2 - 6: If you start with a big number as the radius of the defining circle, your table of sines will be more accurate than if you start with a smaller number. You can use decimal fractions to add more digits (#5 describes decimal fractions which were then a quite new concept). After calculating the table you can remove any decimal fractional digits as the error made in so doing is negligible.

From the Tomash Library on the History of Computing

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh

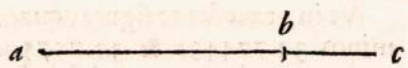
ſunt 9987643 $\frac{8213051}{10000000}$, accipi poſſunt hi 9987643 abſque ſenſibili errore.

7\. *Eſt prætereà alia accurationis formula; quum ſcilicet quantitas ignota, ſeu numero inexplicabilis, inter terminos numerales pluribus vnitatibus non differentes includitur.*

Vt poſitâ Diametro circuli partium 497; quia neſcitur præciſè quot partium ſit ambitus, ideò eruditiores ex Archimedis ſententiâ, eum inter terminos 1562, & 1561 incluſerunt. Item ſi coſtarum quadrati quælibet ſit partium 1000, erit diagonalis radix quadrata numeri 2000000; quæ cùm ſit numero inexplicabilis, ideò per extractionem radicis quadratæ quæruntur ejus termini, ſcilicet 1415 terminus major, & 1414 terminus minor, vel 1414 $\frac{604}{2828}$ major, & 1414 $\frac{604}{2829}$ minor: videlicet quanto minor terminorum differentia ſit, tanto major accuratio.

Vice ipſarum quantitatum ignotarum, termini earum ſunt addendi, ſubſtrahendi, multiplicandi, aut dividendi prout opus fuerit.

8\. *Adduntur bini alicuius quantitatis termini ad binos terminos alterius, quum minor illius minori huius, & maior illius maiori huius additur.*



Vt ſit linea *a b c*, in duas quantitates *a b*, & *b c* diviſa; ſit *a b* inter terminos 123.5 majorem, & 123.2 minorem: ſitque *b c* inter terminos 43.2 majorem, & 43.1 minorem. Additis ergo majore ad majorem, & minore ad minorem, fiet tota quantitas *a c* inter terminos 166.7 & 166.3.

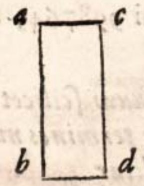
9\. *Multiplicantur bini alicuius quantitatis termini per binos terminos alterius, quum minor illius in minorem huius, & maior illius in maiorem huius ducitur.*

Proposition 7: If a number is unknowable (e.g. a repeating decimal like the square root of 2) then you can assign a lower bound (l) and an upper bound (u) and perform the operations on these bounds rather than on the number itself.

Proposition 8: If doing an addition with two numbers (A and B) that have lower and upper bounds Al, Au, Bl, Bu, then the result will be between $Al + Bl$ and $Au + Bu$.

Proposition 9: If multiplying two numbers with bounds, the result will be between $Al \times Bl$ and $Au \times Bu$

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



Vt ſit altera quantitas *a b*, quæ ſit inter terminos 10.502 majorem, & 10.500 minorem; altera verò *a c*, quæ ſit inter terminos 3.216 majorem, & 3.215 minorem. Vnde ductis 10.502 in 3.216, & 10.500 in 3.215, provenient termini 33.774432, & 33.757500; inter quos erit area *a b c d*.

10. *Terminorum ſubſtractio fit, terminum maiorem minoris quantitatis à minore maioris, & minorem minoris à maiore maioris auferendo.*

Vt in ſchemate primo, ſi ex terminis *a c*, qui ſunt 166.7 & 166.3, ſubduxeris terminos *b c*, qui ſunt 43.2 & 43.1, fient 123.6 & 123.1 pro terminis *a b*; & non 123.5 & 123.2. Nam licet etiam horum additio ad 43.2 & 43.1, producebat 166.7 & 166.3, (per octavam) non tamen è converſo ſequetur, quin aliquid poſſit eſſe inter 166.7 & 166.3, ex quo ſi ſubſtraxeris aliquid quod ſit inter 43.2 & 43.1, remaneret id quod non ſit inter 123.5 & 123.2: verùm non eſſe id inter terminos 123.6 & 123.1 eſt impoſſibile.

11. *Diviſio fit, partiendo terminum maiorem dividendi per minorem diviſoris, & minorem dividendi per maiorem diviſoris.*

Vt in præcedente figura, quadratum *a b c d* inter terminos 33.774432 & 33.757500 conſtitutum dividatur per terminos *a c*, qui ſunt 3.216 & 3.215, provenient $10.505\frac{857}{3215}$ & $10.496\frac{2364}{3216}$ pro terminis *a b*; & non 10.502 & 10.500, eâdem ratione, quam in ſubſtractione diximus.

12. *Rudes terminorum fractiones delendæ ſunt addita vnitate ad terminum maiorem.*

Vt pro terminis *a b* præcedentibus, ſcilicet $10.505\frac{857}{3215}$ & $10.496\frac{2364}{3216}$, capiantur 10.506 & 10.496.

Hacte-

Proposition 10: Subtraction of two bounded numbers will yield a result between (assuming A < B) B*l* - A*u* and B*u* - A*l*.

Proposition 11: In division the bounds for A/B will be between A*u*/B*l* and A*l*/B*u*

Proposition 12: In limits, the fractional parts of both the lower and upper limits may be removed if 1 is added to the upper limit.

Hactenus de accuratione, ſequitur de facilitate operis.

13. *Omnis progreſsionis Arithmeticæ facilis eſt conſtructio, Geometricæ autem non omnis.*

Patet hoc, ſiquidem additione & ſubſtractione fit facillimè Arithmetica progreſsio: geometrica verò, difficillimis multiplicationibus, diviſionibus, & radicum extractionibus continuatur.

Solæ Geometricæ illæ progreſsiones facilè continuantur, quæ per ſubſtractionem facilis partis numeri à numero toto oriuntur.

14. *Partes numeri faciles dicimus, partes quaſlibet cujus denominationes unitate & cyphris quotcunque notantur; habentur autem hæ partes, rejiciendo tot figuras ultimas principalis numeri, quot ſunt cyphræ in denominatore.*

Vt partes decima, centeſima, milleſima, 10000^{a}, 100000^{a}, 1000000^{a}, 10000000^{a}, faciles dicuntur, quia cujuſlibet numeri decima pars habetur delendo ejus ultimam figuram; centeſima duas ultimas, milleſima tres ultimas figuras, & ſic de cæteris, ſemper delendo tot figuras ultimas quot ſunt cyphræ in denominatione partis. Vt decima pars hujus 99321 eſt 9932, ejus autem centeſima eſt 993, milleſima 99, &c.

15. *Mediocriter etiam facilè habentur partes dimidia, vigeſima, ducenteſima, & aliæ per binarium & cyphras denominatæ, rejiciendo tot figuras ultimas principalis numeri, quot ſunt cyphræ in denominatore, & reliquum bipartiendo.*

Vt numeri 9973218045 pars 2000^{a} eſt 4986609, pars 20000^{a} eſt 498660.

16. *Hinc ſequitur, ſi à ſinu toto ſeptem cyphris aucto, cæteriſque inde ortis ſuam* 10000000^{am} *partem ſubſtraxeris, continuari poſſunt quam facillimè centum numeri, in ea proportione Geometrica, quæ eſt inter ſinum totum & ſinum eo minorem unitate, ſcilicet* 10000000 *&* 9999999; *hancque ſeriem proportionalium primam Tabulam nominamus.*

Proposition 13: The construction of an arithmetic series is easy, geometrical series are not always easy.
Proposition 14: To find the tenth, hundredth, thousandth, etc. part of a number, simply remove the last 1, 2, 3, etc. digits.
Proposition 15: A half, twentieth, two-hundredth, etc. part of a number is easily found by removing as many least significant digits are there are zeros in the divisor and then dividing the remaining number by 2.
Proposition 16: A table (the "first table") of 100 numbers begins with 10,000,000.0000000 and subsequent entries can be found by, at each stage, subtracting its 10,000,000 part (i.e., the first entry is 10,000,000; second entry is that number minus 1 = 9,999,999; third entry is 9,999,999 minus 0.9999999 = 9999998.0000001, etc. as shown on the next page. The last of the 100 entries should be 9,999,900.0004950.

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Prima Tabula.

10000000.0000000
1.0000000
9999999.0000000
9999999
9999998.0000001
9999998
9999997.0000003
9999997
9999996.0000006
continuando vsq; ad
9999900.0004950

Vt ex ſinu toto aucto ſeptem cyphris (majoris accurationis gratiâ) ſic 10000000.0000000 aufer 10000000, fient 9999999.0000000: ex quibus aufer 9999999, fient 9999998.0000001; & ſic proſequaris ut à latere, donec centum creaveris proportionalia, quorum ultimum (ſi rectè computaveris) erit 9999900.0004950.

17. *Tabula ſecunda progreditur à ſinu toto ſex cyphris aucto, per quinquaginta numeros alios deficientes proportionaliter, ea proportione quæ facillima eſt, & quàm proxima proportioni, quæ eſt inter primum & vltimum primæ Tabulæ.*

Secunda Tabula.

10000000.000000
100.000000
9999900.000000
99.999000
9999800.001000
99.998000
9999700.003000
99.997000
9999600.006000
&c. vsq; ad
9995001.222927

Vt primæ Tabulæ primum & vltimum ſunt 10000000.0000000, & 9999900.0004950; in quorum proportione, difficile eſt conſtituere quinquaginta proportionales numeros. Proxima itaque facilis proportio, eſt 100000 ad 99999; quæ continuatur ſatis exactè adjiciendo ſinui toti ſex cyphras, & auferendo ab antecedente ſuam partem 100000[am], vt fiat ſubſequens, vt à latere vides: & hæc Tabula præter primum qui eſt ſinus totus, etiam quinquaginta contineat proportionales numeros, quorum vltimum (ni erraveris) reperies eſſe 9995001.222927.

Proposition 17: The second table is constructed much the same as the first, except there are only 6 positions to the right of the decimal point in the starting number, there will be only 50 numbers in this table and the last number must be as close as possible to the last entry in the first table. This can be done by subtracting the 100,000th part of each entry to obtain the next. The last line on this page says that the final entry in the second table should be 9995001.222927 (this is incorrect - it should have been 9995001.224804 - see the Macdonald translation mentioned in the introductory notes for an extensive discussion of how this error propagated through all of Napier's logarithm tables).

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18. *Tertia Tabula ſexaginta novem columnis conſtat, & in qualibet columna ponuntur numeri viginti & vnus, progredientes ea proportione quæ facillima eſt, & quàm proxima illi proportioni quæ eſt inter primum & vltimum ſecundæ Tabulæ.*

Vnde hujus prima columna facillimè habetur à ſinu toto quinque cyphris aucto, & à cæteris inde ortis ſuam 2000^am^ partem auferendo.

Prima Columna tertiæ Tabulæ.

10000000.00000
5000.0000
9995000.00000
4997.50000
9990002.50000
4995.00125
9985007.49875
4992.50374
9980014.99501
&c. vſq; ad
9900473.57808

Vt quia inter 10000000.000000 primum ſecundæ tabulæ, & 9995001.222927 ejuſdem vltimum, proportio difficilis eſt progreſsionis; ideò in proportione facili 10000 ad 9995 (quæ illi propinqua ſatis eſt) conſtituendi ſunt numeri viginti & vnus; quorum vltimus (ni erraveris) erit 9900473.57808. A quibus jam creatis, rejici poteſt vltima ſingulorum figurâ abſque ſenſibili errore, quò faciliùs ab iis alii poſteà creentur.

19. *Primi numeri omnium columnarum, progrediuntur à ſinu toto quatuor cyphris aucto, eâ proportione facillimâ, & proximâ proportioni, quæ eſt inter primum & vltimum primæ columnæ.*

Vt primæ columnæ primus & vltimus ſunt 10000000.0000, & 9900473.5780: his proportio facillima maximè propinqua eſt 100 ad 99. A ſinu igitur toto continuandi ſunt 68 numeri in ratione 100 ad 99, auferendo à quolibet eorum ſuam centeſimam partem.

20. *Eâdem proportione, à primæ columnæ numero ſecundo, per*

 omnium

Proposition 18: A third table is to be constructed of 69 columns. Each column contains 21 entries constructed much like those in the first and second tables. The first column begins with 10,000,000,000.00000 (i.e., 5 digits to the right of the decimal point) and subsequent numbers are found by subtracting the 2,000th of the previous entry as shown.

Once the last entry has been found it should be recorded but then (in subsequent steps) you may disregard any of the least significant digits so that your calculations will be easier.

omnium columnarum ſecundos : & à tertio, per tertios : & à quarto, per quartos : & à cæteris reſpectivè, per cæteros fit progreſsio.

Vt ex antecedentis columnæ numero aliquo fit numerus ejuſdem ordinis in ſequenti columna, ſubſtrahendo ſuam centeſimam partem, numeroſque hoc qui ſequitur ordine conſtituendo.

PROPORTIONALIA TERTIÆ TABVLÆ.

Prima Columna.	*Secunda Col.*
10000000.0000	9900000.0000
9995000.0000	9895050.0000
9990002.5000	9890102.4750
9985007.4987	9885157.4237
9980014.9950	9880214.8451
&c. vſq. ad	&c. deſcendendo ad
9900473.5780	9801468.8423

Tertia Col.	*Inde 4ª. 5ª. &c. vſq. ad 69ᵃᵐ columnæ.*	
9801000.0000	&c. vſque ad	5048858.8900
9796099.5000	&c. vſque ad	5046334.4605
9791201.4503	&c. vſque ad	5043811.2932
9786305.8495	&c. vſque ad	5041289.3879
9781412.6967	&c. vſque ad	5038768.7435
&c. deſcendendo ad		vſque tandem ad
9703454.1539	vſque tandem ad	4998609.4034

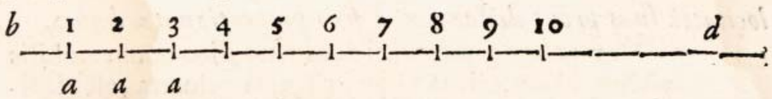
Proposition 19: The first and last entries in column 1 are 10,000,000 and 9,900,473 and the ratio between them is very close to the ratio of 100 to 99. Use this ratio (starting at the first column) to fill in the first entry of each column from 2 to 69, each entry being 0.01 less than the previous one. Similarly, fill in the second entries of each column by making it 0.01 less than the entry in the previous column.

21. *In tertia ergo Tabula, habes inter ſinum totum, & medium ſinus totius, interjectos ſexaginta octo numeros in proportione vt* 100 *ad* 99; *& rurſus inter ſingulos binos horum, interjectos viginti numeros in proportione vt* 10000 *ad* 9995: *& rurſus inter binos primos horum, ſcilicet inter* 10000000 *&* 9995000, *habes in ſecunda Tabula interjectos* 50 *numeros, in proportione vt* 100000 *ad* 99999: *& tandem inter binos primos horum, habes in prima Tabula interiectos centum numeros, in proportione vt* 10000000 *ſinus totus ad* 9999999; *quorum differentia quum ſit tantum vnitatis, non eſt opus eam (interiectis mediis) minutiùs partiri. Vnde hæ tres tabulæ (poſtquam completæ fuerint) ad tabulam † artificialem computandam ſufficient.* † Log rithm

Hucuſque ſinus ſeu numeros naturales proportione geometrica progredientes tabulis facillimè inſerere docuimus.

22. *Supereſt Tabulæ ſaltem tertiæ, apud ſinus ſive numeros naturales geometricè decreſcentes, ſuos † numeros artificiales Arithmeticè creſcentes inſerere.* † Loga rithm.

23. *Arithmeticè creſcere, eſt æqualibus temporibus æquali ſemper quantitate augeri.*



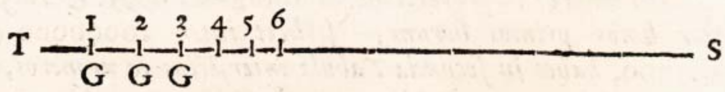
Vt ex puncto *b* fixo verſus *d*, infinitè producatur linea: in qua, ex *b* verſus *d* procedat punctus *a*, movens ea lege, vt æqualibus temporis momentis æqualibus feratur ſpatiis: quæ ſint *b* 1, 1 2, 2 3, 3 4, 4 5, &c. Dico hoc incrementum per *b* 1, *b* 2, *b* 3, *b* 4, *b* 5, &c. Arithmeticum dici. In numeris autem ſint *b* 1, 10: *b* 2, 20: *b* 3, 30: *b* 4, 40: *b* 5, 50. Dico 10, 20, 30, 40, 50, &c. Arithmeticè creſcere: quia æqualibus momentis, æquali numero denarii ſemper augeri intelliguntur.

Proposition 21: The third table now contains numbers from the initial radius of the circle defining the sines (10,000,000,000) to half that value. The numbers in the columns are interpolated in that range by the ratio 100 to 99. Between each value at the head of the columns there are now 21 other values (down the column) in the ratio of 10,000 to 9,995. Similarly for the first and second tables. These three table can be used to produce a table of logarithms.

Proposition 22: The third table contains numbers that decrease geometrically. Now the logarithms of these numbers must be added in arithmetically increasing sequence.

Proposition 23: This defines an arithmetic sequence (see proposition 2)

24. *Geometricè decreſcere, eſt æqualibus temporibus quantitatem primò totam, inde aliam atque aliam ejus partem ſuperſtitem, ſimili ſemper proportionali parte diminui.*



Vt ſit linea ſinus totius T S, in hac moveatur punctus G, à T in 1 verſus S. quantoque tempore defertur à T in 1, quæ ſit (exempli gratiâ) decima pars T S: tanto idem G tempore moveatur ab 1 in 2, quæ ſit decima pars 1 S: & à 2 in 3, quæ ſit decima pars 2 S: & à 3 in 4, quæ ſit decima pars 3 S, & ſic de cæteris. Dico hos ſinus T S, 1 S, 2 S, 3 S, 4 S, &c. dici Geometricè decreſcere: quia inæqualibus ſpatiis proportione ſimilibus & tempore æqualibus diminuuntur. In numeris ſit T S, 10000000: 1 S, 9000000: 2 S, 8100000: 3 S, 7290000: 4 S, ſit 6561000, &c. Dico hos ſinuum numeros, æqualibus temporibus ſimili proportione diminutos, dici Geometricè decreſcere.

25. *Vnde punctus mobilis Geometricè ad fixum accedens, velocitates ſuas prout diſtantias, à fixo proportionatas habet.*

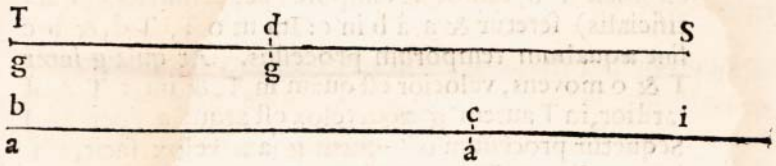
Vt repetito præcedenti Schemate, dico quum mobilis pūctus geometricus G eſt in T, ejus velocitas eſt ut diſtantia T S: & quum G eſt in 1, ejus velocitas eſt ut 1 S: & quum in 2, ejus velocitas eſt ut 2 S, & ſic de cæteris. Atque ita quæ eſt proportio diſtantiarum T S, 1 S, 2 S, 3 S, 4 S, &c. adinvicem, ea etiam erit proportio velocitatum G in punctis T, 1, 2, 3, 4, &c. adinvicem. Nam magis minúſve velox punctus arguitur, prout magis minúſve longè ſub æqualibus temporibus ferri conſpicitur. Qualis itaque proceſſus ratio, talem etiam & velocitatum eſſe neceſſe eſt: at talis eſt ſub æqualibus temporibus ratio proceſſuum T 1, 1 2, 2 3, 3 4, 4 5, &c.

qualis

Proposition 24: This defines a decreasing geometric sequence (see proposition 2)
Proposition 25: Defines a line (illustrated in Proposition 24) and a point moving from T to S with ever decreasing velocity. The velocity decreases in the ratio of the distance remaining to S to the whole distance of the line.

qualis diſtantiarum T S, 1 S, 2 S, 3 S, 4 S, &c. ut mox docebimus. Vnde neceſſariò qualis habitudo diſtantiarum G ab S, videlicet T S, 1 S, 2 S, 3 S, 4 S, &c. invicem; talis etiam eſt velocitatum G in punctis T, 1, 2, 3, 4, &c. quod erat demonſtrandum. At quòd proceſſuum T 1, 1 2, 2 3, 3 4, 4 5, &c. talis ſit ratio, qualis diſtantiarum T S, 1 S, 2 S, 3 S, 4 S, &c. patet: quia quantitatum proportionaliter continuatarum differentiæ etiam in eadem proportione continuantur. At hæ diſtantiæ (per hypotheſin) proportionaliter continuantur, & illi proceſſus ſunt harum differentiæ: quare eâdem proceſſus quâ diſtantias ratione continuari certum eſt.

26. † *Numerus artificialis ſinus dati, eſt qui Arithmeticè crevit tantâ ſemper velocitate, quantâ ſinus totus incepit Geometricè decreſcere, tantoque tempore, quanto ſinus totus in ſinum illum datum decrevit.* † *Logarithm.*



Sit ſinus totus linea T S, ſinus datus in eadem linea d S: certis quibuſdam momentis moveatur g Geometricè à T in d. Sitque alia linea b i verſus i infinita, in qua ex b moveatur a Arithmeticè, eâdem velocitate quâ g primò cùm erat in T: totidemque temporis momentis procedat a ex b fixo verſus i uſque in c punctum: dicetur numerus metiens b c lineam numerus artificialis ſinus d S dati.

7. *Vnde ſinus totius nihil eſt pro artificiali.*

Nam ex Schemate, cùm g eſt in T faciens ſuam diſtantiam ab S ſinum totum, punctus Arithmeticus a incipiens in b, nuſquam inde proceſſit. Vnde ex definitione diſtantiæ, ſinus totius nullus erit artificialis.

Proposition 26: The line TS is the radius of the defining circle for the sine function, so T is assumed to be the whole sine (10,000,000,000) and S is assumed to be the sine of zero degrees (i.e., zero). TS has a point, d, moving down it with decreasing velocity (a decreasing geometric series) while the line bi has a point moving down it with constant velocity (and increasing arithmetic series). The logarithm of the sine dS is the number measuring the line bc.

Proposition 27: The logarithm of the whole sine is zero.

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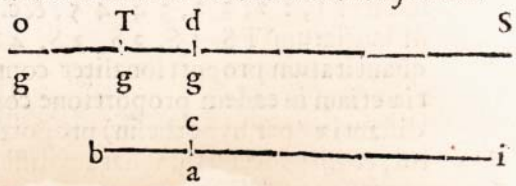
28. *Hinc etiam ſequitur, quod cujuſlibet dati ſinus numerus artificialis, major eſt differentiâ inter ſinum totum, & ſinum datum; & minor differentiâ quæ eſt inter ſinum totum, & quantitatem eo majorem in eadem ratione, quæ eſt ſinus totius ad datum. Atque hæ differentiæ dicuntur ideò termini artificialis.*



Vt repetito præcedenti Schemate, protractâq; lineâ S T ultra T in o, Ita ut S o ſe habeat ad T S, ut T S ad d S. Dico ſinus d S numerum artificicialem b c, majorem eſſe quàm T d, & minorem quàm o T. Quanto enim tempore g ab o in T fertur, tanto & g à T in d feretur (per 24) quia o T eſt tanta pars o S, quanta T d eſt lineæ T S, tantoque tempore (per definitionem artificialis) feretur & a à b in c: Ita ut o T, T d, & b c ſint æqualium temporum proceſſus. At quia g inter T & o movens, velocior eſt quàm in T, & inter T & d tardior, in T autem g æquivelox eſt atque a (per 26.) Sequetur proceſſum o T quem g jam velox facit, majorem eſſe: & T d proceſſum quem g tardus facit, minorem eſſe: & b c proceſſum (quem punctus a mediocri ſuo motu totidem etiam temporis momentis perficit) medium quoddam eſſe inter utrumque, quod erat demonſtrandum. Numeri itaque artificialis quem b c deſignat, dicitur o T terminus major, & T d terminus minor.

29. *Dati itaque ſinus artificiales terminos exhibere.*

Ex præmiſſa probatur minorem terminum relinqui, ablato ſinu dato à ſinu toto; & majorem terminum produci, multiplicato ſinu toto in terminum minorem, & producto diviſo per ſinum datum, ut ſequenti exemplo.

30. *Vnde primæ Tabulæ primum proportionale, quod eſt*

999-

Proposition 28: The logarithm of a sine will always be bounded by an upper and lower limit which can be determined by the geometry of these lines and points (see the translation by Macdonald for details).
Proposition 29 and 30: These explain, and give an example, of how to find the limits between which any given logarithm must fall.

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9999999, *habet ſuum artificialem numerum inter terminos* 1.0000001 *&* 1.0000000.

Nam per præmiſſam aufer 9999999 à ſinu toto cyphris aucto, fiet unitas cum ſuis cyphris pro minore termino: hanc unitatem cyphris auctam, & multiplicatam in ſinum totum, divide per 9999999, & fiet 1 0000001; ſive (ſi majorem accurationem requiris) 1.00000010000001 pro majore termino.

31. *Inſenſibili differentia diſtantes termini ipſi, ſive inter eos quidvis pro numero artificiali vero habetur.*

Vt in ſuperiori exemplo, ſinus hujus 9999999, artificialis numerus habetur hic 1.0000000, ſive hic 1.00000010, ſive omnium optimè hic 1.0000005: quia enim ipſi termini 1.0000001 & 1.0000000, inſenſibili fractione utpote $\frac{1}{10000000}$ differunt ab invicem: ideò & ipſi, & quicquid inter eos eſt, multò minùs, multóque inſenſibiliore errore, à vero different artificiali inter hos terminos conſtituto.

32. *Quotcunque ſinuum Geometricâ proportione à ſinu toto deficientium, vnius artificiali numero aut terminis datis, cæterorum etiam dare.*

Conſequitur hoc neceſſariò incrementi Arithmetici, decrementi Geometrici, & numeri artificialis definitiones: ſiquidem per illas, ut ſinus Geometricâ proportione decreſcunt continuò: ita interim ſui artificiales, continuo Arithmetico progreſſu per æqualia accreſcunt. Vnde cuilibet ſinui Geometricæ progreſſionis decreſcendo, reſpondet ſuus artificialis Arithmeticæ progreſſionis creſcendo: primo ſcilicet primus, & ſecundo ſecundus, & ſic deinceps.

Ita ut ſi primus artificialis, reſpondens primo ſinui poſt ſinum totum detur, ſecundus artificialis erit ejus duplum, tertius triplum, & ſic de cæteris: donec omnes omnium artificiales innoteſcant, ut ſequenti exemplo patebit.

Proposition 31: If the upper and lower limits of a logarithm are very close together, then either of these numbers (or a number between them) may be taken as the logarithm with very little effect on any resulting computation. Proposition 32: In a table of logarithms, if the first one in the table after the radius itself is known, then the others may be found from it simply because they will be part of a uniformly increasing arithmetic sequence. If the second logarithm in the table in *x* then the third will be 2*x*, the next 3*x* etc.

33. *Hinc omnium ſinuum proportionalium primæ Tabulæ, numeri artificiales inter terminos propinquos includi, & per conſequens exactè ſatis dari poſſunt.*

Vt cùm ſinus totius artificialis ſit o (per 27) & primi poſt ſinum totum, qui eſt 9999999 in prima Tabula, artificialis ſit (per 30) inter terminos 1.0000001, & 1.0000000: neceſſariò ſecundi poſt ſinum totum, qui eſt 9999998.0000001, artificialis continebitur inter dupla illorum terminorum: ſcilicet inter 2.0000002 & 2.0000000: & tertii 9999997.0000003, inter eorundẽ tripla: ſcilicet inter 3.0000003 & 3.0000000. Et ſic in cæteris, æqualiter ſemper augendo terminos intervallo primorum terminorum: donec omnium proportionalium primæ Tabulæ artificiales terminos compleveris. Poteris conſimili progreſſu, ſi libet, numeros ipſos artificiales exiguo & inſenſibili errore continuare hoc ordine: pro ſinus totius artificiali, erit o: pro primi poſt ſinum totum artificiali, erit 1.00000005 (per 31:) pro ſecundi, 2.00000010: pro tertii, 3.00000015. Et ita deinceps.

34. *Differentia artificialium ſinus totius & ſinus dati, eſt ipſius dati artificialis.*

Patet hoc, quum enim ſinus totius artificialis ſit nihil per 27, hoc nihilo ex artificiali dati ſubducto, ipſum integrum artificialem dati remanere neceſſe eſt.

35. *Duorum artificialium differentia, addenda eſt ad artificialem maioris ſinus eorundem, vt habeas artificialem minoris: & ſubſtrahenda ab artificiali minoris ſinus, vt habeas artificialem maioris.*

Neceſſariò hoc fit, ſiquidem creſcunt artificiales decreſcentibus ſinibus, atque minor eſt artificialis majoris ſinus, & major minoris. Ideóque æquum eſt differentiam addere minori artificiali, ut habeas artificialem majorem licet minoris ſinus: & contrà, auferre differentiam

tiam

Proposition 33: Napier shows that the limits on second logarithm in the table (for the sine 9,999,999) must be the numbers 1.0000000 and 1.0000001 (the true logarithm for this second sign must be between these two numbers). This means that the logarithm for the third sine (9,999,998.0000001) must be between 2.0000000 and 2.0000002, the logarithm for the fourth sine (9999997.0000003) must be between 3.0000000 and 3.0000003, etc.
Proposition 34: Because the logarithm of the radius is zero, the difference between the logarithms of any other sine and the logarithm of the radius must have the value of the logarithm of that sine (i.e., x - 0 = x).
Proposition 35: As the sines get smaller the logarithms get bigger, thus for two sines A and B (A>B) if log(B) - log (A) = *x* then log(B) = log(A) + *x* and log(A) = log(B) - *x*. Note that this takes some thought for the modern reader as Napier's logarithms were, in some ways, the reverse of modern base 10 logarithms.

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tiam à majore artificiali, ut habeas minorem artificialem licet majoris ſinus.

36. *Similiter proportionatorum ſinuum ſunt æqui-differentes artificiales.*

Conſequitur hoc neceſſariò definitiones artificialium & motuum: Nam cùm per eas, Geometrico decremento ſimiliter proportionato, reſpondet Arithmeticum incrementum æquale ſemper: neceſſariò ſimiliter proportionatis ſinibus, reſpondere æqui-differentes artificiales & numeros, & numerorum terminos concludimus. Vt in ſuperiori exemplo primæ Tabulæ, quia ſimilis eſt proportio inter primum proportionale poſt ſinum totum *9999999*.0000000, & tertium *9999997*.0000003: ei quæ eſt inter quartū *9999996*.0000006, & ſextum *9999994*.0000015. Ideò numerus artificialis iſte 1.00000005 primi, differt ab artificiali iſto 3.00000015 tertii, eâdem differentiâ, quâ artificialis iſte 4.00000020 quarti, differt ab artificiali iſto 6.00000030 ſexti proportionalis. Eâdem etiam eſt æqualitatis ratio inter differentias terminorum artificialium adinvicem: videlicet tam minorum inter ſe, quam etiam majorum inter ſe, quorum ſinus ſunt ſimiliter proportionati.

37. *Vt trium ſinuum in proportione Geometricâ continuatorum, quadratum medii æquatur facto ex ductis invicem extremis: Ita in ſuis artificialibus numeris, duplum medii æquatur aggregato extremorum. Vnde horum artificialium duobus quibuſcunque datis, tertius innoteſcit.*

Quia horum trium ſinuum, ratio quæ eſt inter primum & ſecundum, ſimilis eſt rationi quæ eſt inter ſecundum & tertium: Ideò (per præmiſſam) ſuorum artificialium ea eſt differentia inter primum & ſecundum, quæ eſt inter ſecundum & tertium. Sit (verbi gratiâ) primus artificialis lineâ b c expreſſus, ſecundus lineâ

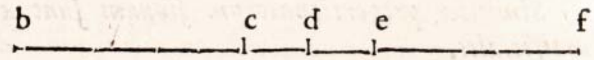
C 2 b d,

Proposition 36: If sines are in the same ratio (i.e., A/B and C/D are the same) then the logarithms of A and B and the logarithms of C and D are the same when subtracted.

Proposition 37: If three numbers (Napier says three sines which amounts to the same thing) A, B and C are in geometric progression, then it is known that $B^2=A*C$ and thus log(B)*2 = log(A)+log(C).

Napier, of course, did not use this modern notation but expressed the relationship in words, as was the custom at the time.

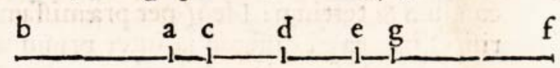
b d, tertius lineâ b e: ſintque unicâ lineâ b c d e comprehenſi hoc modo



ſintque differentiæ c d & d e æquales: horum medium b d dupletur, productâ lineâ hâc à b ultra e in f, ita ut b f ſit duplum b d. Dico b f æquari utriſque lineis, b c primi artificialis, & b e tertii: ab æqualibus enim b d & d f, aufer æqualia c d & d e: ſcilicet c d, à b d, & d e, à d f: & remanebunt b c & e f neceſſariò æqualia. Cùm itaque tota b f, æqualis ſit utriſque b e & e f: ergo & utriſque b e & b c æquabitur, quod erat demonſtrandum. Vnde ſequitur canon: ſi trium horum artificialium medium datum duplaveris, & hinc ſubſtraxeris extremum datum, reliquum extremorum quæſitum innoteſcet: & ſi extrema data conjunxeris, & aggregatum hoc bipartiveris, medium fiet notum.

38. ***Quatuor Geometricè proportionalium, ſicut factum ex ductu mediorum, æquatur facto ex ductu extremorum: Ita ſuorum artificialium, aggregatum mediorum æquatur aggregato extremorum. Vnde horum artificialium tribus quibuſcunque datis, quartum innoteſcit.***

Quia horum quatuor proportionalium, ratio quæ eſt inter primum & ſecundũ, ſimilis eſt rationi quæ eſt inter tertium & quartum: Ideò (per penultimè præmiſſam) ſuorum artificialium, ea eſt differentia inter primum & ſecundum, quæ eſt inter tertium & quartum. Tales itaque quantitates in linea b f ſupraſcripta ſumantur, ut hîc, quarum b a primum artificialem, b c ſecundum, b e tertium, & b g quartum referat, factis differentiis



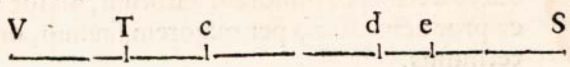
a c & e g æqualibus: Ita ut d in medio c e poſitum, in medio a g etiam poni neceſſe eſt. Iam dico aggregatum

b c

Proposition 38: If any four numbers A, B, C, and D are in some geometric progression, then B*C = A*D and thus log(B) + Log(C) = log(A)+Log(D).

b c ſecundi, & b e tertii : æquari aggregato b a primi, & b g quarti. Nam quia (per præmiſſam) duplum b d, quod eſt b f, æquatur utriſque b c & b e: quia differentiæ eorum à b d, videlicet c d & d e ſunt æquales. Eâdem ratione, & idem b f æquabitur utriſque b a & b g: quia eorum differentiæ à b d, videlicet a d & d g ſunt etiam æquales. Quum itaque & aggregatum ex b a & b g, & aggregatum ex b c & b e, ſint iidem duplo b d, quod eſt b f æqualia: ergo & inter ſe æquabuntur, quod erat demonſtrandum. Vnde ſequitur canon, ſi quatuor horum artificialium, ab aggregato extremorum datorum, ſubduxeris alterum mediorum cognitum, relinquetur reliquum medium quod quærebatur: & ſi ab aggregato mediorum cognitorum ſubduxeris alterum extremorum cognitum, relinquetur extremum quæſitum.

39. *Duorum artificialium differentia, eſt inter duos terminos, ad quorum maiorem ſe habet ſinus totus, vt eorum artificialium minor ſinus ad ſinuum differentiam: & ad minorem terminum ſe habet ſinus totus, vt artificialium ſinus maior ad ſinuum differentiam.*



Sit ſinus totus T S, ſinus duo dati d S major, & e S minor: Vltra S T ſignetur puncto V diſtantia T V, ea lege, ut S T ſe habeat ad T V, ut e S minor ſinus, ad d e differentiam ſinuum. Deinde citra T verſus S, ſignetur puncto c diſtantia T c, ea lege, ut T S ſe habeat ad T c, vt d s ſinus major, ad d e differentiam ſinuum. Dico differentiam artificialium reſpondentium ſinibus d S & e S, conſtitui inter terminos V T majorem, & T c minorem. Nam quia ex hypotheſi, ut e S ad d e, ita T S ad T V; & ut d S ad d e, ita T S ad T c ſe habent: ideò etiam (ex natura proportionalium) ſequuntur duæ concluſiones : Primò, quod V S ſe habet

Propositions 39 and 40 show that the difference between the logarithms of two sines lies between a lower and upper limit and that these limits can be so close that their difference can be ignored, thus giving you the logarithm of the difference between two logarithms of sines. This is simply to say that if you know the logarithms of two sines (A and B), you can easily find the logarithm of A/B.

Details of the exact process can be found in the English translation by Macdonald mentioned in the introductory notes.

ad T S, ut idem T S ad c S. Secundò, quod similis eſt ratio T S ad c S, rationi quæ eſt d S ad e S. Et propterea (per 36) differentia artificialium reſpondentium ſinibus d S & e S, æqualis eſt differentiæ artificialium reſpondentium ſinui toto T S, & ſinui c S. At hæc differentia (per 34) eſt artificialis ipſius ſinus c S: & hic artificialis inter terminos V T majorem, & T c minorem (per 28 poſ.) includitur: quia per primam concluſionem jam dictam, V s major ſinu toto ſe habet ad ſinum totum T s, ut idem T s ad c S. Vnde neceſſariò differentia artificialium reſpondentium ſinibus d S & e S, conſtituitur inter terminos V T majorem, & T c minorem, quod erat demonſtrandum.

40. ***Terminos differentiæ inter artificiales numeros duorum datorum ſinuum exhibere.***

Quum per præmiſſam, ſinus minor ſe habeat ad differentiam ſinuum, ut ſinus totus ad majorem terminum differentiæ artificialium: & ſinus major ſe habeat ad differentiam ſinuum, ut ſinus totus ad minorem terminum: ſequetur ex natura proportionalium, quod ducto ſinu toto per differentiam datorum ſinuum, orietur ex producto diviſo per minorem datorum, maior terminus: & ex producto diviſo per majorem ſinuum, orietur minor terminus.

EXEMPLVM.

VV ſit ſinuum datorum major *9999975*.5000000, minor autem *9999975*.0000300: quorum differentiâ .*4999700* ductâ in ſinum totum (adjectis priùs octo cyphris utrique poſt punctum demonſtrationis gratiâ, licet alioquin ſeptem ſufficiant) quod hinc producitur, ſi per maiorem ſinum, ſcilicet per *9999975*.5000000 diviſeris, provenient .*49997122* octo figurarum poſt punctum pro minore termino. Sin quod producitur, per minorem ſinum, ſcilicet per *9999975*.0000300 diviſeris, provenient .*49997124* pro maiore

ter-

termino: inter quos (ut demonſtratum eſt) conſtituitur differentia artificialium ſinuum datorum. Sed quia protractio huius fractionis in octavam figuram ultra punctum, eſt accuratio pluſquã requiſita, præſertim cùm in ipſis ſinibus ſeptem tantum ponantur figuræ poſt punctum: ideò deletâ octavâ illâ ſive ultimâ utriuſque termini figurâ, uterque terminus unà cum ipſa artificialium differentia, in fractione .4999712 ſtabiliri poteſt, abſque vel minimo ſcrupulo ſenſibilis erroris.

41. *Sinuum vel numerorum naturalium, non in ipſos proportionales primæ Tabulæ, ſed prope vel inter eos cadentium: numeros artificiales, eorúmve ſaltem terminos inſenſibili differentia diſtantes exhibere.*

Sinui dato ſinum primæ Tabulæ proximum, ſive minorem ſive maiorem nota: huius tabulati ſinus terminos artificiales (per 33) quære, & inventos reſerva: deinde (per præmiſſam) terminos differentiæ inter artificiales numeros ſinus dati & ſinus tabulati, ſive ambos, ſive (quia ferè æquales ſunt, ut ſuperiori exemplo patet) eorum alterutrum quære. Hos iam inventos, horúmve alterutrum adde ad illos nuper reſervatos terminos, aut ab illis ſubſtrahe (per 8. 10 & 35.) prout ſinus datus fuerit minor aut maior tabulato ei proximo: & qui hinc producuntur numeri, erunt termini propinqui inter quos includetur artificialis numerus ſinus dati.

EXEMPLVM.

VT ſit ſinus datus 9999975.5000000, cui ſinus in Tabula proximus, eſt 9999975.0000300 minor dato: huius termini artificiales (per 33) ſunt 25.0000025 & 25.0000000: deinde (per præmiſſam) differentia inter artificiales numeros ſinuum dati & tabulati, eſt .4999712: quam (per 35) aufer ab illis terminis, quia ſunt termini minoris ſinus, & provenient 24.5000313 & 24.5000288, termini quæſiti ſinus da-

Proposition 41: This instructs the user that, should they want to find a logarithm of a sine (or natural number) not in the tables, then they can use the process of the last few propositions to determine the value.

ti 9999975.5000000: cuius ipse artificialis numerus in quovis terminorum, sive optimè in 24.5000300 (per 31) constitui potest absque sensibili errore.

ALIVD EXEMPLVM.

SIT sinus datus 9999900.0000000, sinus tabulatus ei proximus 9999900.0004950, huius termini artificiales sunt 100.0000100 & 100.0000000 (per 33:) Deinde differentia inter artificiales numeros illorum sinuum, erit (per præmissam).0004950, quam (per 35) ad superiores terminos adde, & fient 100.0005050 pro maiore termino, & 100.0004950 pro minore termino, inter quos artificialis quæsitus sinus dati includitur.

42. *Hinc sequitur, omnium proportionalium secundæ Tabulæ numeros artificiales exactè satis dari: seu inter terminos cognitos insensibili fractione differentes includi posse.*

Vt in præcedenti exemplo, quum numerus artificialis huius sinus 9999900 (qui est primum proportionale secundæ Tabulæ) demonstretur inter terminos hos 100.0005050 & 100.0004950 esse: necessariò secundi proportionalis, numerus artificialis (per 32) erit inter hos terminos 200.0010100 & 200.0009900: & tertii proportionalis, inter hos terminos 300.0015150 & 300.0014850, &c. Et tandem ultimi sinus secundæ Tabulæ, qui est 9995001.222927, artificialis numerus includetur inter terminos hos 5000.0252500 & 5000.0247500. Quibus iam habitis terminis (per 31) poteris ipsos artificiales numeros eorum exquirere.

43. *Sinuum vel numerorum naturalium non in ipsos proportionales secundæ Tabulæ, sed prope vel inter eos cadentium, numeros artificiales exhibere: seu inter terminos cognitos insensibili fractione differentes includere.*

Sinui dato sinum secundæ Tabulæ proximum, sive maiorem sive minorem nota: huius tabulati sinus terminos

Proposition 42: The logarithms not actually listed in the second table may, from methods demonstrated in proposition 41, be found.
Proposition 43 provides an example of the process.

nos artificiales per præmissam quære: deinde per regulam proportionis, quære quartum proportionale se habens ad sinum totum: ut sinuum dati & tabulati minor se habet ad majorem. Quod uno modo perfici poterit, ducendo dati & tabulati minorem in sinum totum, & productum in majorem dividendo. Altero modo faciliore, ducendo sinuum dati & tabulati differentiam in sinum totum, & productum in dati & tabulati majorem dividendo, atque quotientem ex sinu toto auferendo. At quia hujus quarti proportionalis, numerus artificialis (per 36) tantum differt ab artificiali sinus totius, quantum invicem artificiales sinuum dati & tabulati differunt: Et quia etiam illorum differentia, eadem est cum ipso artificiali quarti per 34: Ideò artificiales terminos quarti, per penultimè præmissam è Tabula prima quære, & inventos adde ad artificiales terminos tabulati, aut ab illis substrahe per 8. 10. & 35. prout tabulatus sinus fuerit major aut minor dato, & producentur artificiales termini sinus dati.

EXEMPLVM.

VT sit sinus datus 9995000.000000, sinus Tabulæ secundæ ei proximus est 9995001.222927, hujus termini artificiales (per præmissam) sunt 5000.0252500 & 5000.0247500. Quartum deinde proportionale alterutro modorum suprascriptorum quære, & fiet 9999998.7764614, cujus terminos artificiales (per 41) è prima Tabula quære, eruntque 1.2235387 & 1.2235386: quos ad superiores terminos per 8 & 35 adde, fientque pro terminis artificialibus dati 5001.2487888 & 5001.2482886. Vnde & numerus inter hos medius, qui est 5001.2485387, optimè (per 31 pos.) pro ipso artificiali numero sinus 9995000 dati statuitur absque sensibili errore.

44. *Hinc sequitur, omnium proportionalium primæ Columnæ tertiæ*

Propositions 44 and 45 show that logarithms for numbers not actually listed in the third table may be determined by following the processes similar to those for the earlier tables.

tertiæ Tabulæ, numeros artificiales exactè satis dari: seu inter terminos cognitos insensibili fractione differentes includi posse.

Nam quum per præmissam, hujus 9995000 (qui est primus sinus infra sinum totum, ex proportionalibus primæ Columnæ tertiæ Tabulæ) numerus artificialis sit 5001.2485387 absque errore sensibili: secundi proportionalis scilicet 9990002.5000, numerus artificialis (per 32) erit 10002.4970774. Et sic in cæteris, progrediendo usque ad ultimum ejus columnæ sinum 9900473.57808: cujus, pari ratione artificialis numerus erit 100024.9707740: eiusque termini 100024.9657720 & 100024.9757760 erunt.

45. *Numerorum naturalium, seu sinuum non in ipsos proportionales primæ Columnæ tertiæ Tabulæ, sed prope vel inter eos cadentium, numeros artificiales exhibere: seu inter cognitos terminos insensibili fractione differentes includere.*

Sinui dato sinum primæ Columnæ tertiæ Tabulæ proximum, sive minorem sive maiorem nota; huius tabulati terminos artificiales per præmissam quære: deinde quartum proportionale se habens ad sinum totum, ut sinuum dati & tabulati minor ad maiorem, per unum ex modis in penultimè præcedente descriptis quære: huius quarti ita inventi terminos artificiales (per penultimè præmissam) è secunda Tabula quære, & inventos adde ad terminos tabulati sinus superiùs inventos, aut ab illis substrahe (per 8. 10. & 35.) & producentur artificiales termini sinus dati.

EXEMPLUM.

VT sit sinus datus 9900000, proportionalis sinus primæ Columnæ tertiæ Tabulæ ei proximus, est 9900473.57808, cuius termini artificiales per præmissam sunt 100024.9657720 & 100024.9757760. Quartum inde proportionale erit 9999521.6611850, cuius

Proposition 45 provides an example of the process described in proposition 44.

cuius termini artificiales (per 43 è ſecunda Tabula deſumpti) ſunt 478.3502290 & 478.3502812: quibus terminis ad terminos ſuperiores tabulati (per 8 & 35) additis, provenient termini 100503.3260572 & 100503.3160010, inter quos neceſſariò cadit artificialis numerus quæſitus. Vnde numerus inter hos medius qui eſt 100503.3210291, pro vero artificiali numero ſinus 9900000 dati, ſtatui abſque ſenſibili errore poteſt.

46. *Hinc ſequitur, omnium proportionalium tertiæ Tabulæ numeros artificiales exactè ſatis dari.*

Nam quum (per præmiſſam) 100503.3210291, ſit artificialis primi ſinus ſecundæ Columnæ, qui eſt 9900000, cæterique primi reliquarum columnarum ſinus eadem proportione progrediantur; neceſſariò (per 32 & 36) eorum numeri artificiales eadem ſemper differentia creſcunt, additis 100503.3210291 antecedenti artificiali, ut fiat ſequens. Habitis ergo ſic primis artificialibus cuiuſque columnæ, atque per penultimè præcedentem omnibus artificialibus primæ columnæ datis; elige tibi, an mavis ſimul eiuſdem columnæ omnes artificiales condere, addendo ſemper ad ſuperiorem artificialem cuiuſlibet columnæ, hanc artificialium differentiam 5001.2485387, ut fiat proximè inferior eiuſdem columnæ artificialis: An mavis ſimul eiuſdem ordinis omnes artificiales, ſcilicet omnes ſecundos ſingularum columnarum artificiales; inde omnes tertios, inde quartos, & ſic reliquos conſtituere, addendo ſemper 100503.3210291 cuilibet artificiali præcedentis columnæ, ut eiuſdem ordinis ſequentis columnæ artificialis proveniat. Vtrovis enim modo, omnes omnium huius Tabulæ proportionalium habentur artificiales; quorum ultimus, & ad ſinum 4998609.4034 congruens, eſt 6934250.8007528.

47. *Omnibus tertiæ Tabulæ naturalibus numeris, aſcribendi ſunt ſui artificiales, ut tertia Tabula integra fiat & perfecta:*

Proposition 46: This indicates that all logarithms for the numbers in the third table may be found with sufficient accuracy to not be a factor in any subsequent calculation.

quam poſthàc ſemper radicalem vocabimus.

Hæc hujus Tabulæ conſcriptio fit conſtituendo columnas numero & ordine quibus per 20 & 21 deſcribuntur: & diviſâ unaquâque columnâ in duas ſeries;

RADICALIS TABVLÆ

Columna prima.		*Columna ſecunda.*	
Naturales.	Artificiales	Naturales.	Artificiales
10000000.0000	0	9900000.0000	100503.3
9995000.0000	5001.2	9895050.0000	105504.6
9990002.5000	10002.5	9890102.4750	110505.8
9985007.4987	15003.7	9885157.4237	115507.1
9980014.9950	20005.0	9880214.8451	120508.3
&c. uſq. ad	uſque ad	uſque ad	uſque ad
9900473.5780	100025.0	9801468.8423	200528.2

Columna 69ª.

	Naturales.	Artificiales.
& cæteri uſque ad	5048858.8900	6834225.8
	5046334.4605	6839227.1
	5043811.2932	6844228.3
	5041289.3879	6849229.6
	5038768.7435	6854230.8
	& tandem	uſque ad
	4998609.4034	6934250.8

Proposition 47: Add the logarithms (artificial numbers) to the third table (he now terms this the *radical table*) and gives examples of what the first, second and last column will contain.

quarum prima, proportionalia illa Geometrica, quæ ſinus numeroſque naturales nominamus; ſecunda, hos ſuos artificiales Arithmeticè per æqualia progredientes contineat. Duobus tamen (compendii gratiâ) animadverſis: Primò, quòd illis omnibus artificialibus, unam poſt punctum relinqui figuram ſatis ſit, cæteris ſex noviſſimis jam rejectis: quas tamen ſi initio neglexiſſes: error inde frequenti multiplicatione priorum tabularum, accreviſſet in hac tertia intollerabilis. Secundò, ſi ſecunda poſt punctũ figura excedat quaternarium: figura prima, quæ ſola poſt punctum relinquitur, eſt unitate augēda. Vt pro 10002.48, &c. rectius eſt ponere 10002.5, quàm 10002.4: & pro 1000 35001, aptiùs ponimus 1000.4, quàm 1000.3. Itaque eo ſitu procedat jam radicalis Tabula quo præmittitur.

48. *Perfectâ jam radicali Tabulâ, ex ea ſola Tabulæ artificialis numeros excerpimus.*

Vt enim priores duæ Tabulæ ad conſtitutionem tertiæ inſerviebant; Ita tertia hæc radicalis ad principalem artificialem Tabulam, quàm facillimè & abſque errore ſenſibili condendam inſervit.

49. *Sinuum majorum quàm 9996700, artificiales numeros facillimè exhibere.*

Fiet hoc, ſola ſubſtractione ſinus dati à ſinu toto. Nam per 29, artificialis numerus ſinus 9996700, eſt inter terminos 3300 & 3301; qui quidem termini (quia invicem unitate tantum differunt) à ſuo artificiali vero, non poſſunt errore ſenſibili, videlicet majore unitate differre. Vnde ipſe terminus minor 3300, qui ſola ſubſtractione habetur, pro ipſo artificiali capi poteſt. Eadem neceſſariò ratio eſt de omnibus ſinibus hoc majoribus.

50. *Sinuum omnium intra limites Tabulæ radicalis comprehenſorum, artificiales exhibere.*

Sinuum dati, & tabulati ei proximi, differentiam

Proposition 48: Now that the radical table is finished we can take the logarithms of numbers (or sines) from it alone.

Proposition 49: To find logarithms of sines greater than 9996700 simply subtract it from the radius because (by proposition 29) log(9996700) must be between 3300 and 3301 and thus we can take the lower limit (3300) as the required logarithm. The process works for all sines greater than this example.

duc in sinum totum; productum partire per facillimum divisorem, qui vel sit sinus datus, vel tabulatus ex proximus, vel inter utrumque utcunque constitutus; & producetur differentiæ artificialium aut terminus major, aut minor, aut intermedium quidpiam (per 39) quorum nullus à vera artificialium differentia errore sensibili differet, propter propinquitatem numerorum Tabulæ. Et ideò hunc eorum quemcunque productum (per 35) adde, ad artificialem tabulati in Tabula repertum, si sinus datus sit minor tabulato sinu: alioquin illum productum ex hoc tabulati artificiali substrahe, & proveniet dati sinus numerus artificialis quæsitus.

EXEMPLVM.

VT, sit sinus datus 7489557, cujus quæritur artificialis. Sinus tabulatus ei proximus est 7490786.6119, hinc aufero illam adjectis cyphris sic 7489557.0000, relinquentur 1229.6119; quæ ducta in sinum totum, divido per numerum facillimum, qui sit vel 7489557.0000, vel 7490786.6119; vel optimè per quippiam inter eos constitutum, utpote per 7490000, & facillima divisione provenient 16401: quæ (quia datus sinus minor est tabulato) adde ad artificialem tabulati, videlicet ad 2889111.7, & fient 2890751.8, quæ idem valent quod 2890751$\frac{4}{5}$: sed quia Tabula principalis nec fractiones admittit, nec quicquam ultra punctum, ponimus pro illo 2890752, qui est artificialis quæsitus.

ALIVD EXEMPLVM.

SIt sinus datus 7071068.0000; sinus tabulæ ei proximus erit 7070084.4434, quorum differentia est 983.5566; quibus ductis in sinum totum, productum divide optimè per 7071000, quæ sunt inter sinus datum & tabulatum, provenient inde 1390.9: quæ (quia sinus datus excedit tabulatum ei proximum) substrahatur ex artificiali numero tabulati in tabula reperto,

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to, ſcilicet à 3467125.4, remanebit 3465734.5. Vnde 3465735 ponitur pro artificiali quæſito ſinus 7071068 dati. Itaque hæc libertas diviſorem eligendi miram parit facilitatem.

51. *Omnes ſinus in proportione dupla, habent 6931469.22 pro differentia ſuorum artificialium.*

Quia enim omnis ſinus ad ſuum dimidium eadem eſt ratio, quæ eſt ſinus totius ad 5000000: ideò (per 36) differentia artificialium cujuſque ſinus & ſui dimidii, eſt eadem cum differentia artificialium ſinus totius, & ſui dimidii 5000000. At eadem eſt differentia artificialium ſinus totius, & ſinus 5000000, cum ipſo artificiali ſinus 5000000 (per 34) cuius 5000000, artificialis (per præmiſſam) erit 6931469.22. Ergo & idem numerus 6931469.22 erit differentia omnium artificialium, quorum ſinus ſunt in proportione dupla: & per conſequens duplum ejus, ſcilicet 13862938.44, erit differentia omnium artificialium, quorum ſinus ſunt in ratione quadrupla: & triplum ejus, videlicet 20794407.66, erit differentia omnium artificialium, quorum ſinus ſunt in ratione octupla.

52. *Omnes ſinus in proportione decupla, habent 23025842.34 pro differentia ſuorum artificialium.*

Nam per penultimè præmiſſam, ſinus 8000000 habet artificialem ſuum 2231434.68: & per præmiſſam, differentia inter artificiales ſinuum 8000000, & ſuæ octavæ partis 1000000, eſt 20794407.66: Vnde per additioné fiunt 23025842.34, pro artificiali ſinus 1000000: & quum ad hunc ſinus totus ſit decuplus, omnes ſinus in ratione decupla, eandem illam differentiam 23025842.34, inter ſuos artificiales habebunt, eâdem cauſâ & ratione, quam jam in dupla proportione per præcedentem expoſuimus, quod probandum erat. Et per conſequens, centuplæ proportioni reſpondebit hujus artificialis duplum, quod eſt 46051684.68, pro dif-

Propositions 51 to 52 are comments on the methods of interpolating between values in the third table.

differentia artificialium: Et ejuſdem triplum, quod eſt 69077527.02, erit differentia omnium artificialium, quorum ſinus ſunt in ratione millecupla. Et ſic de ratione 10000^{a}, & aliis, ut infrà.

53. *Vnde omnes ſinus in ratione compoſita ex duplo & decuplo, habent artificiales ſuos differentiâ 6931469.22, & differentiâ 23025842.34 reſpectivè differentes.*

Vt in tabella ſubſequenti conſpicere licet.

Sinuum proportiones datæ.	Artificialium reſpondentes differentiæ.	Sinuum proportiones datæ.	Artificialium reſpondentes differentiæ.
Dupla	6931469.22	8000^{pla}	89871934.68
Quadrupla	13862938.44	10000^{pla}	92103369.36
Octupla	20794407.66	20000^{pla}	99034838.58
Decupla	23025842.34	40000^{pla}	105966307.80
20^{cupla}	29957311.56	80000^{pla}	112897777.02
40^{cupla}	36888780.78	100000^{pla}	115129211.70
80^{cupla}	43820250.00	200000^{pla}	122060680.92
Centupla	46051684.68	400000^{pla}	128992150.14
200^{pla}	52983153.90	800000^{pla}	135923619.36
400^{pla}	59914623 12	1000000^{pla}	138155054.04
800^{pla}	66846092.34	2000000^{pla}	145086523.26
Millecupla	69077527.02	4000000^{pla}	152017992.48
2000^{pla}	76008996.24	8000000^{pla}	158949461.70
4000^{pla}	82940465.46	10000000^{pla}	161180896.38

54. *Omnium ſinuum ultra limites radicalis Tabulæ excluſorum, numeros artificiales inveſtigare.*

Hoc facilè fit, ſinum datũ multiplicando per 2, 4, 8, 10, 20, 40, 80, 100, 200: vel per alium quemvis proportionis numerum hac tabella expreſſum, donec producatur numerus, qui intra limites radicalis tabulæ contineatur

Napier constructs another table contain logarithms that can be used in the process described in Proposition 54.

Proposition 54: if a sine falls outside the limits of the radical table, then multiply it be 2, 4, 8, 10, 20 etc (the numbers from the first and third columns of the table in the previous proposition) until it falls within the limits of the radical table numbers. Select the logarithm of the resulting number (or the nearest one to it) from the radical table and add the logarithm (columns 2 and 4) from the table.

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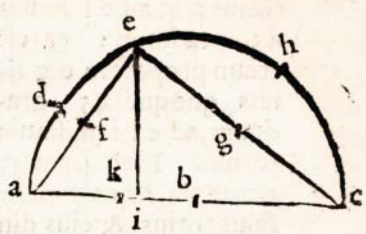
tineatur. Hujus jam sub Tabula comprehensi artificialem (per 50) quære, cui acquisito adde tandem differentiam artificialem, quam Tabella indicat priori convenisse multiplicationi.

EXEMPLVM.

QVæratur, quem artificialem sinus 378064 habeat; is cùm ultra limites Tabulæ radicalis excludatur, per numerum aliquem proportionum præcedentis tabellæ, utpote per 20 ducatur, fietque 7561280, cujus jam intra Tabulam cadentis artificialem (per 50) quære, fietque 2795444.9, ad quem adde differentiam in Tabella inventam convenientem vigecuplæ proportioni, quæ est 29957311.56, fientque 32752756.4. Vnde 32752756 est artificialis quæsitus, sinus 378064 dati.

55. *Vt dimidium sinus totius, se habet ad sinum dimidii alicujus arcus; Ita sinus complementi ejusdem dimidii, ad sinum totius arcus.*

Sit sinus totus a b, dupletur & sit a b c; hac diametro fiat semi-circulus, in quo signetur arcus ille a e, bifariam in d divisus: ejus ergo dimidii quod est d e, extendatur complementum ab e versus c, quod fit arcus e h, cui & h c necessariò æquatur: quia d e h quadrans æquatur reliquo quadranti arcuum a d & h c. Proinde ducantur linea e i perpendicularis ad a i c, quæ ideò sinus est arcus a d e: & linea



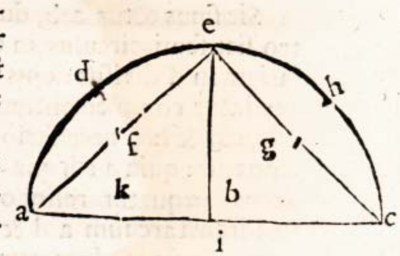
a e, cujus dimidium f e, est sinus arcus d e, qui est dimidium arcus a d e: & linea e c, cujus dimidium e g est sinus arcus e h, & ideò est sinus complementi arcus d e: dimidium autem sinus totius a b sit a k. Dico ut a k se habet ad e f, ita e g ad e i se habebit: duo enim triangu-

Propositions 55, 56 and 57 provide some trigonometric identities that can help simplify the calculations to find a logarithm of certain sines.

guli c e a, & c i e, æqui-anguli ſunt: quia i c e, vel a c e angulus utrique communis eſt, & uterque c i c, & c e a rectus eſt, ille ex hypotheſi, hic, quia in peripheria eſt, & ſemi-circulum occupat. Ideoque ut a c hypotenuſa trianguli c e a, ad eius minus latus a e; ita ſe habet c e hypotenuſa triang. c i e, ad ejus minus latus e i. Et quum totum a c ſe habeat ad a e, ut totum e c ad e i: ſequetur inde dimidium a c, quod eſt a b, ſe habere ad a e, ut dimidium e c, quod eſt e g, ſe habet ad e i. Et denique cùm jam totum a b eſt ad totum a e, ut e g ad e i: Concludimus neceſſariò dimidium a b, quod eſt a k, ſe habere ad dimidium a e, quod eſt f e: ut e g ſe habet ad e i, quod erat demonſtrandum.

56. *Duplum artificialis arcus 45 graduum, eſt artificialis dimidii ſinus totius.*

Repetito præcedenti Schemate ſit caſus talis, quòd a e, & e c, ſint æquales. In hoc caſu cadet i in b, eritque e i ſinus totus, atque e f, & e g æquabuntur: eorumque quivis ſinus eſt 45 graduum. Et quia (per præcedentem) quæ eſt proportio dimidii ſinus totius a k, ad e f ſinum 45 graduum: ea eſt etiam proportio e g ſinus quoque 45 graduum, ad e i iam ſinum totum.



Ideò (per 37) duplum artificialis ſinus 45 graduum, æquale eſt artificialibus extremorum, ſcilicet ſinus totius, & eius dimidii. At horum amborum artificiales, ſunt tantum artificialis alterius eorum, ſcilicet dimidii ſinus totius: quia reliqui ſcilicet ipſius ſinus totius (per 27) artificialis nullus eſt. Neceſſariò igitur duplum artificialis arcus 45 graduum, eſt artificialis dimidii ſinus totius, quod erat demonſtrandum.

57. *Aggregatum ex artificiali dimidii ſinus totius, & artificiali*

ciali cuiusque arcus, æquatur aggregato artificialium dimidii ejus arcus, & complementi hujus dimidii. Vnde artificialis huius dimidii arcus haberi potest, cæterorum trium artificialibus datis.

Quia per penultimè præmissam, dimidium sinus totius proportionatur ad sinum dimidii alicuius arcus, ut sinus complementi eiusdem dimidii arcus, ad sinum totius arcus: Ideò (per 38) aggregatum artificialium duorum extremorum, scilicet artificialis dimidii sinus totius, & artificialis cuiusvis totalis arcus, æquabitur aggregato artificialium mediorum, videlicet artificialis dimidii eiusdem arcus, & artificialis complementi huius dimidii. Vnde & per eandem 38, si addideris artificialem dimidii sinus totius (per 51, vel per præmissam inventum) ad artificialem cuiusvis totalis arcus datum: & hinc substraxeris artificialem complementi dimidii prioris arcus datum, relinquetur ipse artificialis petitus eiusdem dimidii arcus: quæ erant demonstranda.

EXEMPLVM. Sit artificialis dimidii sinus totius (per 51) *6931469*, sitque arcus totalis *69* graduum & 20 minutorum, cuius artificialis sit *665143* datus: totalis arcus dimidium est 34 graduum & 40 minutorum, huius artificialem quæro. Complementum huius dimidii arcus est 55 graduum, & 20 minutorum, cuius artificialis sit 1954370 datus: Addo itaque *6931469* ad *665143*, & fiet aggregatum 7596612: ex quo aufero 1954370, & relinquentur 5642242 artificialis quæsitus, arcus 34 graduum & 40 minutorum.

58. *Datis artificialibus omnium arcuum non minorum 45 gradibus, omnium arcuum minorum artificiales facillimè habentur.*

Ex artificialibus arcuum omnium non minorum 45 gradibus per hypothesin datis, habebis per præmissam, artificiales reliquorum omnium arcuum decrescentium

Proposition 58: Once the logarithms of sines greater than 45 degrees are know then the ones less than 45 degrees are easy to find. Using proposition 57 it is possible to find all the logarithms for sines down to 22 degrees 30 minutes, and from these down to 11 degrees 15 minutes, etc.

usque ad vigesimum secundum gradum cum semisse. Ex quibus iam habitis, artificiales similiter reliquorum arcuum usque ad 11 gradus & 15 minuta habebuntur. Et ex his rursus, artificiales omnium arcuum usque ad 5 gradus & 38 minuta. Et ita deinceps in primum usque minutum.

59. *Tabulam Artificialem condere.*

Paginæ præparentur quadraginta quinque longiusculæ, ut præter margines superiorem & inferiorem, sexaginta etiam lineas numerales capere valeant. Paginarum quælibet lineamentis transversis in 20 spatia æqualia dividatur: spatiorum quodvis tres lineas numerales capere valeat. Inde aliis lineis descendentibus dividatur pagina quævis in columnas septem, interposita duplici linea inter columnas secundam & tertiam, & inter quintam & sextam: inter cæteras verò simplex ponatur linea. Prima pagina in fronte superiore lævorsum, supra tres primas columnas superscribatur hoc titulo | 0 *Gradus* | & subscribatur inferiùs & dextrorsum sub tribus ultimis columnis sic | 89 *Gradus* | Secunda pagina superscribatur lævorsum sic | 1 *Gradus* | & subscribatur dextrorsum sic | 88 *Gradus* | Tertia pagina superscribatur sic | 2 *Gradus* | & subscribatur sic | 87 *Gradus* | Et ita cum cæteris paginis procedendo, ut suprà scripti infrà scriptis additi, quadrantem uno minus sive 89 gradus semper compleant. Inde prima columna per singulas paginas titulum hunc suprascriptum habeat | *Minuta graduum suprascriptorum* | Secunda columna hoc titulo superscribatur | *Sinus arcuum sinistrorum* | Tertia columna hoc titulo superscribatur | *Artificiales arcuum sinistrorum* | Quarta columna hoc titulo & superscribatur & subscribatur | *Differentia inter artificiales complementorum* | Quinta columna subscribatur hac subscriptione | *Artificiales arcuum dextrorum* | Sexta columna subscribatur hac subscriptione | *Sinus arcuum dextrorum* | Septima columna subscribatur hac subscriptione

Proposition 59: Napier describes the layout of the tables in his *Descriptio* (see the file of that publication for an example).

ne | *Minuta gradaum infra ſcriptorum* | Primæ deinde columnæ inſerantur numeri minutorum ab o ad 60 progrediendo. Septimæ etiam columnæ inſerantur numeri minutorum à 60 ad o decreſcendo: ea lege, ut primæ & ſeptimæ columnæ bina quævis minuta in eadem linea oppoſita, gradum integrum ſeu 60 minuta perficiant. Exempli gratia, o ad 60, & 1 ad 59, & 2 ad 58, & 3 ad 57, &c. opponantur. Atq; inter bina quæque viginti lineamentorum tranſverſorum, tres numeri in quolibet intervallo cujuſlibet columnæ contineantur. In ſecunda columna ponantur numeri ſinuum, reſpondentium gradibus ſuprà, & minutiis à latere lævorſum in eadem linea poſitis. In ſexta etiam columna ponantur numeri ſinuum, reſpondentium gradibus infrà, & minutiis à latere dextrorſum in eadem linea poſitis. Hos ſinus ſuppeditabit tibi communis ſinuum REINHOLDI Tabula, vel ſi qua exactior. His peractis, omnium ſinuum inter ſinum totum & ſuum dimidium, artificiales per 49 & 50: cæterorum verò ſinuum artificiales per 54 computato. Sive aliter, multoque & exactiùs & faciliùs, omnium ſinuum inter ſinum totum & ſinum 45 graduum artificiales, per eaſdem 49 & 50 computato: ex quibus jam habitis, omnes reliquorum arcuum minorum 45 gradibus artificiales, per præmiſſam quàm facillimè acquires. Quibus omnibus artificialibus utcunque computatis, in tertia columna locabis artificiales numeros reſpondentes gradibus ſuprà, & minutiis à latere ſiniſtro, ſuiſque ſinibus lævorſum in eadem linea poſitis. Similiter & in quinta columna locabis numeros artificiales reſpondentes gradibus infrà, & minutiis à latere dextro, ſuiſque ſinibus dextrorſum in eadem linea poſitis. Media tandem columna ſic perficitur: numerum quemque artificialem dextrum, ex artificiali ſiniſtrorſum in eadem linea poſito aufer, notatâ differentiâ in eadem linea inter utrumque, donec totam mediam columnam compleveris. Hanc Tabulam nos ad ſingula minuta computavimus, atque eruditis (quibus plus sit otii) ejus exactiorem elimationem, ut & Tabulæ sinuum emendationem relinquimus.

Epitome Tabulæ artificialis aliter condendæ.

60. QUIA *nonnunquam artificiales per* 54 *inventi, differunt ab artificialibus per* 58 *inventis; vt hujus sinus* 378064, *numerus artificialis per illam est* 32752756, *per hanc verò est* 32752741; *arguitur quibusdam in locis Tabula sinuum vitiosa esse. Quapropter consulo eruditis (quibus forsan discipulorum & computistarum copia sit) vt Tabulam sinuum exactiorem & maioris numeri edant, vtpote cuius sinus totus sit* 100000000, *scilicet octo cyphrarum præter vnitatis figuram, cùm prior sinus totus septem tantum constet. Deinde vt Tabula nostra prima contineat centum numeros, progredientes in ea proportione, quæ est inter hunc novum sinum totum, & sinum eo minorem vnitate, vtpote inter* 100000000, *&* 99999999.

Secunda Tabula contineat etiam centum numeros, in ea proportione, quæ est inter hunc novum sinum totum, & numerum eo minorem centenario, scilicet inter 100000000, *&* 99999900.

Tertia Tabula quæ & radicalis dicitur, trigintaquinque columnas, & centum numeros in qualibet columna continet. Centum numeri eiusdem columnæ progrediantur in ea proportione, quæ est decem millium, ad numerum eo minorem vnitate, videlicet 100000000 *ad* 99990000. *Triginta quinque primi inter se, aut secundi, aut tertii, aut cæteri eiusdem ordinis omnium columnarum inter se progrediuntur ea proportione, quæ est* 100 *ad* 99, *aut sinus totius* 100000000 *ad* 99000000. *In*

his

After finishing his 59 propositions, Napier points out that, because there are two different processes that could be used (propositions 54 and 58) to calculate a logarithm, they might differ slightly (which, in fact they do). He suggests that, if you start with a table of sines that are more accurate than his (he suggests using a radius of 100,000,000,000 rather than his radius of 10,000,000,000) the problem will be eliminated.

his ſuiſque artificialibus inveniendis & continuandis, obſerventur regulæ cæteræ præcedentes. Atque ex completa ſic radicali Tabula, omnium ſinuum inter ſinum totum & ſinum 45 graduum artificiales, exactiſſimè per 49 & 50 reperies: atque ex artificiali arcus 45 graduum duplato, habebis artificialem dimidii ſinus totius per 56. Et tandem ex his iam habitis, cæteros artificiales per penultimè præcedentem exquires; quos in ordinem Tabulæ per præcedentem rediges, & fiet Tabula, omnium certè Mathematicarum Tabularum præſtantiſſima & ad vſus præclariſſimos parata.

Finis conſtructionis Tabulæ Artificialis.

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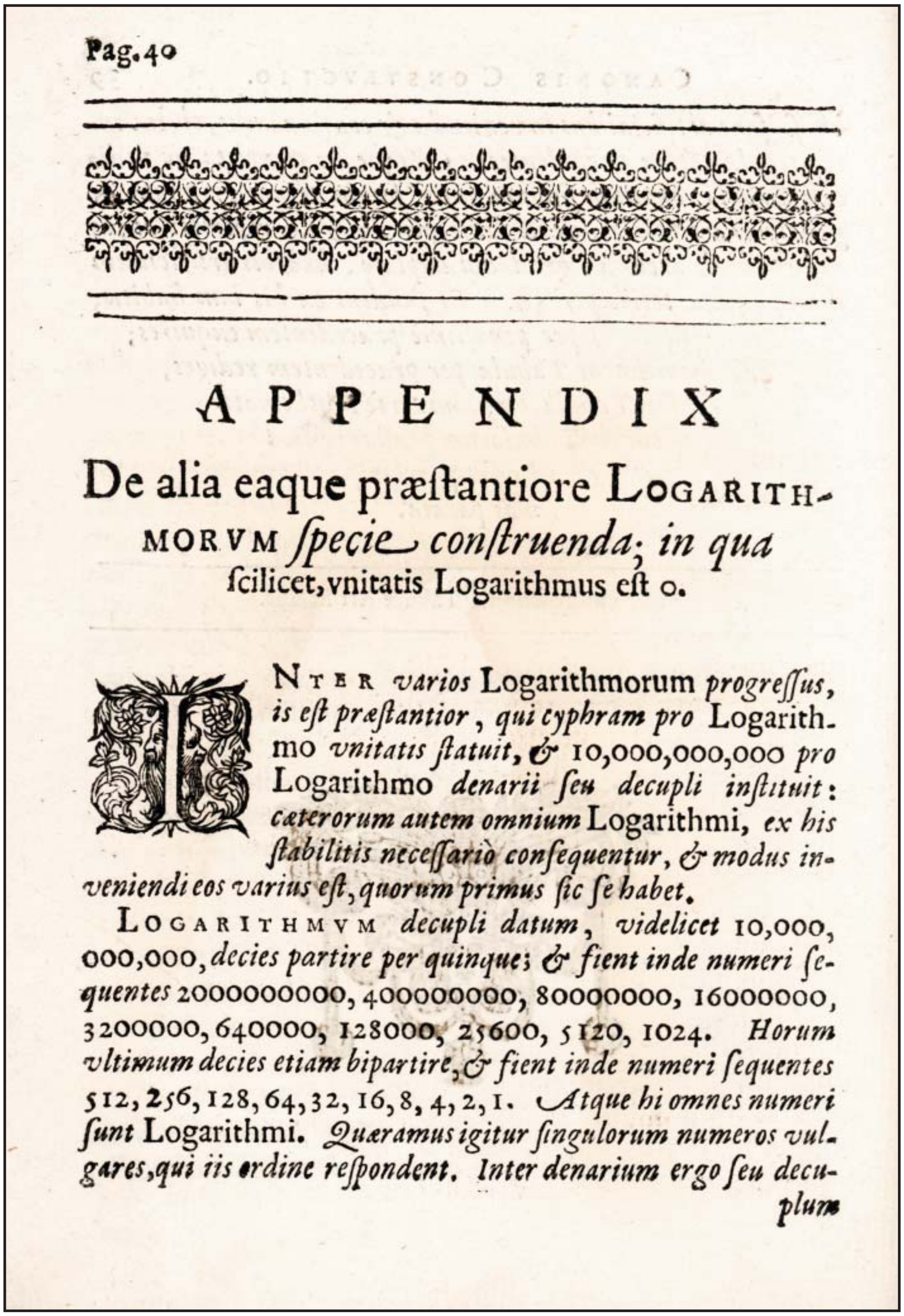

APPENDIX

De alia eaque præſtantiore LOGARITHMORVM *ſpecie conſtruenda; in qua* ſcilicet, vnitatis Logarithmus eſt 0.

INTER *varios* Logarithmorum *progreſſus, is eſt præſtantior, qui cyphram pro* Logarithmo *vnitatis ſtatuit, &* 10,000,000,000 *pro* Logarithmo *denarii ſeu decupli inſtituit: cæterorum autem omnium* Logarithmi, *ex his ſtabilitis neceſſariò conſequentur, & modus inveniendi eos varius eſt, quorum primus ſic ſe habet.*

LOGARITHMVM *decupli datum, videlicet* 10,000,000,000, *decies partire per quinque; & fient inde numeri ſequentes* 2000000000, 400000000, 80000000, 16000000, 3200000, 640000, 128000, 25600, 5120, 1024. *Horum vltimum decies etiam bipartire, & fient inde numeri ſequentes* 512, 256, 128, 64, 32, 16, 8, 4, 2, 1. *Atque hi omnes numeri ſunt* Logarithmi. *Quæramus igitur ſingulorum numeros vulgares, qui iis ordine reſpondent. Inter denarium ergo ſeu decuplum*

This appendix is the one in which Napier describes the base 10 logarithms that were developed by he and Henry Briggs. In this (base 10) system he proposes that the log(1) = 0 and log(10) = 10,000,000,000. Once again the English translation by Macdonald mentioned in the introductory notes should be consulted for the details of the creation of this new form of logarithms.

plum 10 *atque vnitatem (auctos calculi gratiâ quot vis cyphris, vtpote duodenis) capiantur quatuor media proportionalia, seu potiùs (per extractionem radicis supersolidæ) eorundem minimum, quod sit doctrinæ gratiâ* A. *Inter* A *& vnitatem, capiatur similiter ex quatuor proportionalibus minimum medium, quod sit* B. *Inter* B *& vnitatem, capiatur medium quartum seu minimum, quod sit* C. *Et ita progredere per extractionem supersolidæ radicis, dividendo intervallum inter recens inventum & vnitatem, in quinque intervalla proportionalia seu in quatuor media; quorum omnium quartum seu minimum semper notetur, vsque dum ad decimum medium minimum perveneris; quæ his notis signentur* D, E, F, G, H, I, K. *Computatis jam exactè hisce proportionalibus, perge, & inter* K *& vnitatem quære medium proportionale, quod sit* L. *Sic inter* L *& vnitatem cape medium proportionale, quod sit* M. *Sic simile medium inter* M *& vnitatem, quod sit* N. *Eodem artificio (per extractionem quadratam) creentur inter quemque recentem numerum & vnitatem, reliqua intermedia proportionaliâ, his notis signanda* O, P, Q, R, S, T, V: *Quorum proportionalium cuilibet, respondet ordine suus* Logarithmus *superioris seriei. Vnde vnitas erit* Logarithmus *numeri* V, *quicunque is fuerit; & 2 erit* Logarithmus *numeri* T, *& 4 numeri* S, *& 8 numeri* R, 16 *numeri* Q, 32 *numeri* P, 64 *numeri* O, 128 *numeri* N, 256 *numeri* M, 512 *numeri* L, 1024 *numeri* K: *Quæ omnia ex superiore constructione patent. Ex his autem jam constructis, construi possunt aliorum tum* Logarithmorum *proportionalia, tum proportionalium* Logarithmi. *Nam sicuti in staticis ex additione ponderum vnitatis, binarii, quaternarii, 8rii, & aliorum pariter parium numerorum, omnis creari potest ponderum numerus, qui apud nos jam* Logarithmi *sunt: Ita ex proportionalibus* V, T, S, R, *&c. quæ illis respondent, & ex cæteris etiam duplicatâ ratione creandis, con-*

stitui possunt omnium Logarithmorum *oblatorum responden-tia proportionalia, per eorundem invicem multiplicationem re-spectivè, vt docebit experientia. Hujus autem operis præcipua difficultas, est in denis proportionalibus duodecim figurarũ è sex-aginta figuris supersolido more extrahendis: sed quanto major hæc difficultas, tanto exactior est hic modus in* Logarithmis *proportionalium, &* Logarithmorum *proportionalibus inve-niendis.*

Alius modus facilè creandi LOGARITHMOS numerorum compositorum, ex datis LOGARITHMIS suorum primorum.

SI *duo numeri datorum* Logarithmorum, *invicem multi-plicati componunt tertium; eorum* Logarithmorum *aggre-gatum erit tertii* Logarithmus.

Item si numerus per numerum divisus producit tertium, è primi Logarithmo *secundi substractus, relinquit tertii* Loga-rithmum.

Si ex numero in se quadratè, cubicè, supersolidè, &c. ducto, producitur alter quivis; ex primi Logarithmo *duplato, tripla-to, aut quintuplato, producitur illius alterius* Logarithmus.

Item si ex dato per extractionem quadratam, cubicam, su-persolidam, &c. extrahatur radix; datique Logarithmus *bise-cetur, trisecetur, aut per quinque secetur, producetur* Logarith-mus *ejusdem radicis.*

Denique quicunque numerus vulgaris ex vulgaribus compo-nitur per multiplicationem, divisionem, aut extractionem: ejus Logarithmus *componitur respectivè per additionem, substra-ctionem, duplationem, seu triplationem, &c. suorum* Logarith-morum. *Vnde sola difficultas est in numerorum primorum*

Lo-

Logarithmis *inveniendis; qui hac ſequenti arte generali inveniuntur.*

Ad omnes Logarithmos *inveniendos, oportet duorum aliquorum vulgarium numerorum* Logarithmos *dari, aut ſaltem aſſumi pro fundamento operis, vt in ſuperiore prima conſtructione,* 0 *ſeu cyphra aſſumebatur pro* Logarithmo *vulgaris vnitatis, &* 10,000,000,000 *pro* Logarithmo *denarii ſeu* 10. *His itaque datis, quæratur quinarii (qui primus numerus eſt)* Logarithmus *hoc modo. Inter* 10 *&* 1 *quæratur medium proportionale, quod eſt* $\frac{316227766017}{1000000000000}$. *Sic inter* 10,000,000,000 *&* 0 *quæratur medium Arithmeticum, quod eſt* 5,000,000,000. *Deinde inter* 10 *&* $\frac{316227766017}{1000000000000}$ *capiatur medium Geometricum, quod eſt* $\frac{5623413251903}{1000000000000}$. *Et ſimiliter inter* 5,000,000,000 *&* 0 *capiatur medium Arithmeticum, quod eſt* 7500000000.

In continuè proportionalibus vniverſis.

VT *ſumma mediorum & alterutrius extremi, ad eundem extremum; ſic differentia extremorum, ad differentiam extremi ejuſdem & medii proximi.*

Compendium dimidii Tabulæ LOGARITHMORUM.

D*Vorum arcuum quadrantem complentium, vt ſinus majoris, ad ſinum dupli arcus; Ita ſinus* 30 *graduum, ad ſinum minoris. Vnde addito* Logarithmo *dupli arcus ad* Logarithmum 30 *graduum; & à producto, ſubducto* Logarithmo *majoris, relinquitur* Logarithmus *minoris.*

Habitudines LOGARITHMORVM & suorum naturalium numerorum invicem.

1. D*Entur duo ſinus & ſui* Logarithmi. *Si totidem numeri æquales ſinui minori in ſe ducantur, quot ſunt vnitates in majoris* Logarithmo: *& contrà, totidem æquales ſinui majori in ſe ducantur, quot ſunt vnitates in minoris* Logarithmo; *erunt duo producta æqualia, & producti ſinus* Logarithmus, *erit numerus factus ex ambobus* Logarithmis *invicem multiplicatis.*

2. *Vt ſinus major ad minorem; Ita velocitas incrementi, aut decrementi* Logarithmorum *apud minorem, ad velocitatem incrementi aut decrementi* Logarithmorum *apud majorem.*

3. *Duo ſinus in ratione duplicata, triplicata, quadruplicata, &c. habent ſuos* Logarithmos *in ratione dupla, tripla, quadrupla, &c.*

4. *Et duo ſinus in ratione vt ordo ad ordinem, (id eſt vt triplicatum ad quintuplicatum, vel cubus ad ſuperſolidum) habent ſuos* Logarithmos, *in ratione vt eorundem ordinum indices, id eſt, vt 3 ad 5.*

5. *Si primus ſinus in ſecundum ductus producit tertium;* Logarithmus *primi additus ſecundi* Logarithmo *producit tertii* Logarithmum. *Sic in diviſione, diviſoris* Logarithmus *ex dividendi* Logarithmo *ſubductus, relinquit quotientis* Logarithmum.

6. *Et ſi quot æquales primo, invicem ducti producunt ſecundum; totidem æquales primi* Logarithmo, *ſimul additi producunt* Logarithmum *ſecundi.*

7. *Medium quodvis Geometricum inter duos ſinus, habet ſuum* Logarithmum *medium tale Arithmeticum inter ſinuum* Logarithmos,

8. *Sinus primus dividit tertium, quoties ſunt vnitates in* A; *numerus ſecundus dividit eundem tertium, quoties ſunt vnitates in* B: *Item idem primus dividit quartum, quoties ſunt vnitates in* C; *& idem ſecundus dividit eundem quartum, quoties ſunt vnitates in* D. *Dico, quæ eſt ratio* A *ad* B, *eadem eſt* C *ad* D, *&* Logarithmi *ſecundi ad* Logarithmum *primi.*

9. *Hinc fit quod numeri oblati* Logarithmus, *eſt numerus locorum ſeu figurarum, quas comprehendit factum ex oblato toties in ſe ducto quoties ſunt vnitates in* 10,000,000,000.

10. *Item ſi index ordinis ſit* Logarithmus *denarii, numerus figurarum (vnâ demptâ) ordinis ſcilicet multipli, erit* Logarithmus *radicis.*

Quæritur, quis numerus ſit LOGARITHMVS binarii. Reſpondeo, numerus locorum numeri facti ex 10,000,000,000 binariis invicem ductis.

At dices, hic numerus factus ex 10,000,000,000 binariis invicem ductis eſt innumerabilis. Reſpondeo, numerus tamen locorum ejus (quem quæro) eſt numerabilis. Ex data itaque radice (binario) & indice (10,000,000,000) quære numerum locorum multipli, & non numerum ipſius multipli; & per regulam noſtram invenies 3010-29995 &c. pro numero locorum quæſito, & LOGARITHMO binarii.

FINIS.

From the Tomash Library on the History of Computing

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh

LVCVBRATIONES ALIQVOT DOCTISSIMI *D. HENRICI BRIGGII* In APPENDICEM præmiſſam.

Habitudines LOGARITHMORVM & ſuorum naturalium *numerorum invicem; Si vnitatis* LOGARITHMVS ſit 0.

DATIS *duobus numeris cum ſuis* Logarithmis; *ſi communis aliquis diviſor vtroſque* Logarithmos *diviſerit, & vterque numerorum datorum toties in ſeipſum ducatur, vt numerus factorum ab alterutro, vnitate tantum ſuperetur à quoto alterno* Logarithmi; *erunt duo producti æquales. Et* Logarithmus *numeri producti, erit numerus continuè factus, à quotis* Logarithmorum *& communi eorundem diviſore.*

		Logarithmi.
Sunto dati numeri	25118865	4
	39810718	6

In this section Henry Briggs adds notes about the base 10 logarithms. They are rather obscure points, but would have been useful in calculating a table of the base 10 logarithms.

Sit communis divisor vnitas.

Primus in seipsum quinquies } ductus facit 251188649
Secundus in seipsum ter --- } 1000000

				Logarithmi.
Continuè proportionales	1	(0)		0
	25118865	(1)	Latus	4
	63095737	(2)	Quadratus	8
	158489331	(3)	Cubus	12
	39810718	(4)	Biquadratus	16
	100000000	(5)	Solidus	20
	251188649	(6)	Quadr. cubus	24

			Logarithmi.
Continuè proportionales	1	(0)	0
	39810718	(1)	6
	158489331	(2)	12
	630957379	(3)	18
	251188649	(4)	24

ALIVD EXEMPLVM.

		Logarithmi.
Sunto dati numeri	316227766	5
	50118724	7

Communis Logarithmorum divisor sit 1

Primus sexies } in seipsum ductus facit { 316227766
Secundus quater }

		Logarith.			Log.
1	(0)	0	1	(0)	0
316227766	(1)	5	50118724	(1)	7
1000000000	(2)	10	251188649	(2)	14
100	(4)	20	630957376	(4)	28
1000	(6)	30	316227766	(5)	35
316227766	(7)	35			

Notandum si communis diviſor ſit vnitas, vt in vtroque exemplo præcedente; factus ab ipſis datis Logarithmis, *eſt* Logarithmus *numeri producti. Quia vnitas multiplicans non auget multiplicatum.*

Tertivm Exemplvm.

		Logarithmi.	Quoti.
Sunto dati numeri	343	2.53529412	3
	823543	5.91568628	7
Sit communis diviſor		84509804	

Notarum numerus.			
	1	(0)	0
3	343	(1)	2.53529412
6	117649	(2)	5.07058824
8	40353607	(3)	7.60588236
11	3841287201	(4)	10.14117648
18	558545864083284007	(7)	17.74705884
6	823543	(1)	5.91568628
12	678223072849	(2)	11.83137256
18	558545864083284007	(3)	17.74705884

Datorum Logarithmorvm quoti ſunt 3. 7. factus ab iis 21. qui

qui ductus in communem diviſorem 84509804 facit 17. 74705884 LOGARITHMVM numeri producti.

Notandum quod Cubus ſecundi numeri, eique æqualis ſeptimus figuratus primi; (quem aliqui appellant ſecundum ſolidum) ſcribitur notis octodecim: idcirco ejus Logarithmus *in fronte gerit 17. præter notas ſubſequentes, quæ exprimunt* Logarithmum *numeri, qui iiſdem notis ſcribitur: ſed ejus prima tantum nota verſus ſiniſtram, denotat nobis integras vnitates quinque, reliquæ notæ ſubſequentes, exprimunt partes, integris hiſce adjiciendas. Sic* $5\frac{5854586408}{10000000000}$, *&c. cuius* Logarithmus 74705884.

Quod ſi quatuor loci relinquantur integris, ponenda erit in fronte Logarithmi, *nota 3. Sic* $5585\frac{4586408}{10000000}$, *&c. cujus* Logarithmus 3.74705884.

Hinc poterimus datis duobus Logarithmis *& ſinu primi, invenire ſinum ſecundi.*

Sumatur communis aliquis Logarithmorum *diviſor (qui quò major fuerit eò commodior erit) is dividat vtrumque: deinde primus ſinus ſeipſum multiplicet, & ſuos factos: donec numerus factorum vnitate tantum ſuperetur à quoto ſecundi* Logarithmi: *vel donec procreetur figuratus, cognominis quoto ſecundi* Logarithmi. *Idem numerus produceretur, ſi ſecundus ſinus quæſitus, ſeipſum multiplicaret, donec fieret figuratus, cognominis quoto primi* Logarithmi. *Vt patet per præcedentem propoſitionem. Huius itaque figurati, à primi quoto denominati latus quæratur: quod, vbi inventum fuerit, erit ſinus ſecundus quæſitus. Eritque continuè factus à quotis, & communi diviſore, ipſius figurati* Logarithmus.

Vt ſunto dati LOGARITHMI 8, 14, & ſit ſinus primi 3 communis LOGARITHMORVM diviſor eſt 2, qui dat quotos 4.7. Si 3 ſeipſum ſexies multiplicet, proveniet 2187, pro figurato, qui in ſerie continuè proportionalium ab unitate, ſeptimum locum occupabit; & inde dici poterit non incommodè ſeptimus figuratus. Idem numerus 2187, in alia continuè proportionalium ſerie, eſt ab unitate figuratus quartus: cujus latus $6\frac{838521}{1000000}$ eſt ſinus ſecundus quæſitus.

Quoti 4.7. factus ab iis 28. qui ductus in communem diviſorem 2 facit 56. LOGARITHMVM figurati 2187.

			Logar.
Continuè proportionales	1	(0)	0
	3	(1)	8
	9	(2)	16
	27	(3)	24
	81	(4)	32
	243	(5)	40
	729	(6)	48
	2187	(7)	56

		Log.
1	(0)	0
6838521	(1)	14
46765372	(2)	28
31980598	(3)	42
2187	(4)	56

Notandum hos Logarithmos *diverſos eſſe ab iis, qui ad illuſtrationem ſuperioris Propoſitionis adhibebantur; in hoc autem conveniunt, quod vtrobique* Logarithmus *vnitatis eſt* 0. *quo poſito*, Logarithmi *eorundem numerorum vel ſunt æquales, vel ſaltem proportionales inter ſe.*

Sinus primus dividit tertium.) debet primus dividere tertium, & tertii quotum, & quoti deinceps quotum quemlibet, quoties poterit, donec quotus vltimus ſit minor diviſore. Deinde diviſionum harum numerus notetur, non autem quoti alicuius quantitas, (niſi fortè minimi, de quo mox plura dicemus) eodem modo ſecundus, eundem tertium ejuſque quotos dividat. Ita etiam dividatur ab vtroque quartus. Vt

Sunto

Sunto ſinus	primus	2
	ſecundus	4
	tertius	16
	quartus	64

Primus 2, *dividit tertium* 16, *quater. Suntque quoti* 8. 4. 2. 1. Secundus 4, *dividit eundem tertium* 16, *bis. Suntque quoti* 4. 1. *erunt igitur*, A, 4. B, 2.

Eodem modo primus 2, *dividit quartum* 64, *ſexies. Quotique ſunt* 32. 16. 8. 4. 2. 1.

Secundus 4, *dividit quartum* 64, *ter. Quotiq; ſunt* 16. 4. 1. *Sunt igitur* C, 6. D, 3. *aio vt* A, 4. *ad* B, 2: *ſic* C, 6. *ad* D, 3. *& ſic* Logarithmus *ſecundi, ad* Logarithmum *primi.*

Si in hiſce diviſionibus, vltimus & minimus quotus vbique ſit vnitas, vt in iſtis quatuor propoſitis: erunt numeri quotorum, & Logarithmi *diviſorum, reciprocè proportionales. Aliàs ratio non erit prorſus eadem vtrobique: veruntamen ſi diviſores fuerint exigui & dividendi ſatis magni, ita vt quoti ſint plurimi; defectus iſte proportionalium, vix aut ne vix quidem percipi poterit.*

Hinc fit quod numeri oblati Logarithmus) *Sumantur duo numeri* 10. *&* 2, *vel quivis alius; & ſit* Logarithmus *primi datus, ſcilicet* 100, *quæritur* Logarithmus *ſecundi, Primò, ſecundus ſeipſum toties multiplicet, vt numerus factorum, vnitate tantum ſuperetur, à dato primi* Logarithmo. *Deinde vltimus factus, dividatur per primum numerum* 10, *quoties fieri poterit; & eodem modo per ſecundum.* Erit *autem numerus quotorum, facti à ſecundo diviſi,* 100. (*quia factus iſte eſt figuratus centeſimus. Et ſi numerus* *aliquoties in ſeipſum ductus, faciet aliquem: idem numerus factum toties dividet, & ſemel vlterius. vt* 3 *in ſeipſum quater ductus, facit* 243. *idem* 3, *dividit* 243 *quinquies, & quoti erunt* 81.27.9.3.1.) *Deinde ſi idem factus dividatur à primo* 10. *manifeſtum eſt, numerum quotorum, vnitate tantum minorem eſſe numero locorum in diviſo. Idcircò cùm idem factus dividatur à datis duobus numeris, quoties fieri poteſt; erunt (per præcedentem propoſitionem) numeri quotorum, &* Logarithmi *diviſorum, reciprocè proportionales. Eſt autem numerus quotorum ſecundi, æqualis* Logarithmo *primi: idcircò numerus quotorum primi (id eſt numerus locorum in facto, vno dempto,) æquabitur* Logarithmo *ſecundi.*

NVMERI LOCORVM SEV NOTARVM.

	1	0
1	2	1
1	4	2
2	16	4
3	256	8
4	1024	10
7	1048576	20
13	1099511627776	40
25	1208925819614	80
31	1267650600228	100
61	16069379676	200
121	25822496318	400
241	66680131608	800
302	107150835165	1000
603	114813014767	2000
1205	131820283599	4000
2409	17316587168	8000
3011	19950583591	10000

Hic videmus, ſi LOGARITHMVS denarii ſit 10, notæ ſeu loci in decimo figurato ſunt quatuor. Idcirco LOGARITHMVS binarii erit 3 & ampliùs. In centeſimo figurato numerus notarum eſt 31: in milleſimo, 302: in 10000, 3011; & quò plures fuerint facti, eò propiùs acceditur ad verum LOGARITHMVM quæſitum: in minoribus enim factis partes ultimo quoto adhæreſcentes rationes perturbant aliquantulum. Verùm ſi ponatur LOGARITHMVS denarii, eſſe 10,000,000,000; Et binarius in ſeipſum toties ducatur, ut factorum numerus, unitate tantum ſuperetur â dato LOGARITHMO: erit numerus locorum in ultimo facto demptâ unitate, LOGARITHMVS binarii ſatis accuratus; quia particulæ ultimo quoto adjectæ, in numeris adeò magnis, fruſtra conabuntur proportionem impedire.

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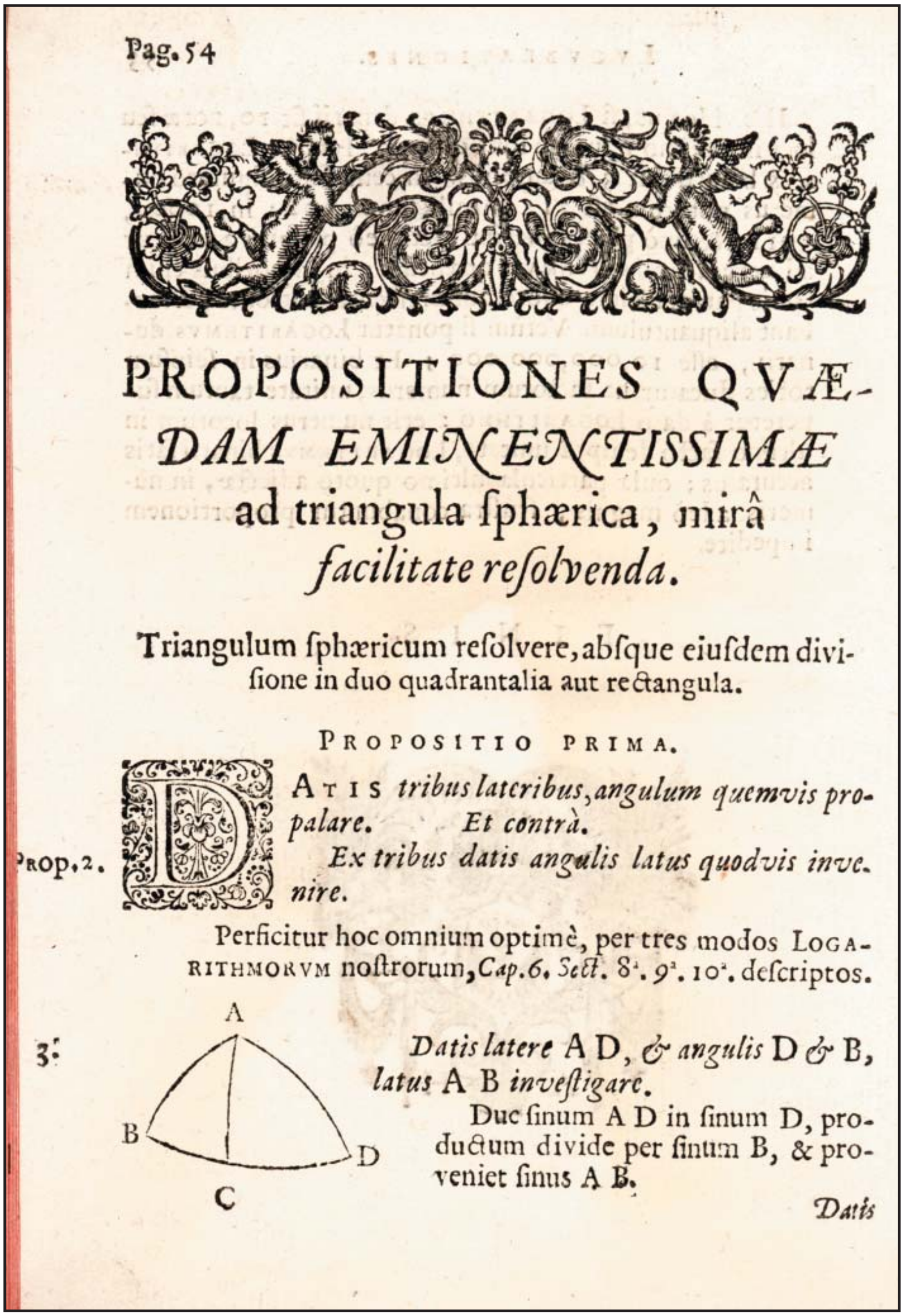

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PROPOSITIONES QVÆDAM EMINENTISSIMÆ ad triangula ſphærica, mirâ *facilitate reſolvenda.*

Triangulum ſphæricum reſolvere, abſque eiuſdem diviſione in duo quadrantalia aut rectangula.

PROPOSITIO PRIMA.

Prop. 2. DATIS *tribus lateribus, angulum quemvis propalare. Et contrà.*

Ex tribus datis angulis latus quodvis invenire.

Perficitur hoc omnium optimè, per tres modos LOGARITHMORVM noſtrorum, *Cap. 6. Sect.* 8ª. 9ª. 10ª. deſcriptos.

3. *Datis latere* A D, *& angulis* D *&* B, *latus* A B *inveſtigare.*

Duc ſinum A D in ſinum D, productum divide per ſinum B, & proveniet ſinus A B.

Datis

This section is devoted to problems involving spherical triangles. These were most often encountered in navigation and astronomy and proved a major stumbling block to many engaged in those professions. The method normally used for solving for the various sides and angles of a triangle drawn on a sphere was to subdivide the triangle with a line (such as AC above) which would create two triangles with at least one 90 degree angle. The various rules for the solution of these right angles spherical triangles were the ones usually used in solving problems. Napier presents a list of 12 formula for solving these problems without subdividing the spherical triangle.

These formula were, as Napier notes, previously published in the *Descriptio*.

4. *Datis latere* A D, *& angulis* D *&* B, *latus* B D *acquirere.*

Duc ſinum totum in ſinum complementi D, & divide per tangentem complementi A D, & fiet tangens C D arcus: deinde duc ſinum C D, per tangentem D, & divide productum per tangentem anguli B, & fit ſinus B C; adde aut ſubſtrahe B C & C D, & fit B D.

5. *Datis latere* A D, *& angulis* D *&* B, *angulum* A *invenire.*

Duc ſinum totum in ſinum complementi A D, & divide per tangentem complementi D anguli, & proveniet tangens complementi C A D; & ſic habetur ipſe C A D angulus. Similiter duc ſinum complementi B anguli, per ſinum C A D, & divide per ſinum complementi D, & fit ſinus anguli B A C; quo addito vel ſubſtracto ex C A D proveniet B A D quæſitus.

6. *Datis* A D, *&* D *angulo cum latere* B D, *invenire angulum* B.

Duc ſinum totum in ſinum complementi D, & divide per tangentem complementi A D, & fiet tangens C D; cujus arcum C D aufer (vel aliàs adde) à latere B D, & fit B C. Deinde duc ſinum C D, per tangentem D, & divide productum per ſinum B C, & fit tangens anguli B.

7. *Datis* A D, *&* D *angulo cum latere* B D, *invenire latus* A B.

Duc ſinum totum in ſinum compleméti D, & divide productũ per tangentem complementi A D, & fiet tangens C D: cuius arcum C D, aufer vel adde lateri B D dato, & fit B C. Deinde duc ſinum complementi A D, per ſinum complementi B C, productum divide per ſinum complementi C D, & proveniet ſinus complementi A B: & ita ipſe A B habetur.

Sequi videtur, ex A D & D angulo cum latere B D datis, invenire angulum A ſeu B A D: ſed hic ſitus triplicem requireret Regulam TRIUM. Mutato igitur A pro B, & B pro A, erit problema ſic. *Datis* B D *&* D, *cum latere* A D, *invenire angulum* B. Quod prorſus idem eſt cum ſeptimo problemate, & duplici tantum regula Trium expeditur.

Datis A D *& angulo* D, *& latere* A B, *angulum* B *invenire.*

Duc ſinum A D in ſinum D, & productum divide per ſinum A B, & producitur ſinus anguli B.

9\. *Datis* A D, *& angulo* D, *& latere* A B, *latus* B D *invenire.*

Duc ſinum totum in ſinum complementi D, & divide productum per tangentem complementi A D, & fiet tangens C D arcus. Deinde duc ſinum complementi C D, in ſinum complementi A B, & productum partire per ſinum complementi A D, & proveniet ſinus complementi B C. Ipſiuſmet ergo B C & C D arcuum ſumma, vel differentia, eſt latus B D quæſitum.

10\. *Datis* A D, *& angulo* D *cum latere* A B, *angulum* A *ſeu* B A D *invenire.*

Duc ſinum totum in ſinum complementi A D, productum divide per tangentem complementi D, & proveniet tangens complementi C A D; & ſic habetur ipſe C A D angulus. Deinde duc tangentem A D, per ſinum complementi anguli C A D, productum divide per tangentem A B, & proveniet ſinus complementi B A C; & inde B A C ipſe: cujus, & C A D arcuum ſumma, vel differentia, eſt B A D angulus quæſitus.

11\. *Datis* A D, *& angulo* D, *cum angulo* A, *latus* A B *exquirere.*

Duc ſinum totum in ſinum complementi A D, & divide productum per tangentem complementi D anguli, & proveniet tangens complementi C A D, & ſic habetur ipſe C A D angulus: cujus, & integri anguli A differentia, (vel aliàs ſumma) eſt angulus B A C. Deinde duc tangentem A D, in ſinum complementi C A D, productum partire per ſinum complementi B A C, & proveniet inde tangens A B.

12\. *Datis* A D *& angulo* D, *cum angulo* A, *angulum tertium* B *invenire.*

Duc ſinum totum in ſinum complementi A D, & divide productum per tangentem complementi anguli D, &

pro-

proveniet ſinus complementi anguli B, & inde ipſe angulus B quæſitus.

Sequi videtur, ex A D, & D, & A angulis, invenire B D latus: ſed in hoc ſitu triplicem requirit regulam Trium. Mutatis igitur A in D, & D in A, erit problema ſub hac forma. *Datis* D A, *&* A, *&* D *angulis, invenire* B A. Prorſus idem cum problemate 11. & duplici tantum Regula Trium expeditur.

De ſemi-ſinuum verſorum præſtantia & vſu.

1. DATIS *duobus lateribus & angulo intercepto, tertium latus invenire.*

Semi-ſinum verſum differentiæ crurum, aufer ex ſemi-ſinu verſo aggregati crurum: reliquum multiplica per ſemi-ſinum verſum anguli verticalis intercepti: & producto diviſo per ſinum totum, adde ſemi-ſinum verſum differentiæ crurum, & prodibit ſemi-ſinus verſus baſis optatæ.

Eâdem ratione ex baſi & angulis juxta eam, reperitur tertius angulus verticalis.

2. *Contrà ex tribus lateribus invenire angulum quemvis.*

Ex ſemi-ſinu verſo baſis, aufer ſemi-ſinum verſum differentiæ crurum in ſinum totum ductum; reliquum divide per ſemi-ſinum verſum aggregati crurum, minutum ſemi-ſinu verſo differentiæ crurum: & prodibit ſemi-ſinus verſus anguli verticalis quæſiti. Eâdem ratione ex tribus angulis inveſtigantur latera.

3. *Datis duobus arcubus tertium dare, cujus ſinus æquetur differentiæ ſinuum priorum.*

Sit arcus 38 : í, ejus Logarithmus 484504: arcus alter 77 gr. Horum accipe complementa 51 : 59, & 13 gr. quorum ſemi-aggregatum eſt 32 : 29, ſemi-differentia verò eſt 19 : 29 : quorum Logarithmi ſunt 621656 &

H 109.

Napier now adds a few more rules using half versed sines (essentially the cosine of half the angle - see the introductory notes on the definition of trigonometric functions).

1098014; quos adde, fient 1719670; à quo producto substrahe 693147, & remanebit 1026523 Logarithmus 21 gr. vel idcirca. Dico sinum rectum 21 gr., qui est 358368, æqualem esse differentiæ sinuum arcuum 77, & 38: í; qui sinus sunt 974370, & 615891 plùs minùs.

4. *Dato arcu, dare Logarithmum ejus sinus versi.*

Sit arcus 13 gr., cujus dimidium 6:30; ejus Logarithmus 2178570, cujus duplum est 4357140: à quo aufer 693147, & remanebit 3663993, cujus arcus est 1:28, & numerus inter sinus positus est 25595: atque is est sinus versus quæsitus 13 gr. ⁂

5. ***Datis duobus arcubus tertium dare, cujus sinus æquetur aggregato sinuum priorum arcuum.***

Sit unus arcus 38:í, alter arcus 1:28: eorum aggregatum est 39:29, & eorum differentia est 36:33: semi-aggregatum autem est 19:44½, semi-differentia verò est 18:16½. Adde ergo Logarithmum semi-aggregati, qui est 1085655, ad Logarithmum differentiæ, qui est 518313, & fit productum 1603968: à quo aufer Logarithmum semi-differentiæ, qui est 1160177, remanent 443791 Logarithmus: cui respondet arcus 39:56, sinus verò 641896. Qui quidem sinus æquatur utriq. sinui 38:í, qui est 615661: & sinui 1:28, qui est 25595 aut juxtà.

6. *Dato arcu & Logarithmo sui sinus recti; arcum dare, cujus sinus versus sit priori sinui recto æqualis.*

Sit arcus 39:56, cui respondet Logarithmus 443791 (ignoto sinu recto,) Logarithmo 443791 adde Logarithmum 693147, fient 1136938. Logarithmum hunc bipartire, & fiet Logarithmus 568469: cujus arcum 34:30 duplica, & fient inde 69 gr. arcus qui quærebatur. Dico enim quod sinus rectus 39 gr. & 56, est æqualis sinui verso 69 gr.: uterque enim sinus est 641800, aut propè.

Trianguli Sphærici A B D, *datis cruribus & angulo verticali, basin dare.*

SIT Triangulum Sphæricum A B D, detur angulus verticalis A, 120 gr. 24 49: crus alterum ambientium detur 34, crus reliquum 47 gr. dimidium anguli verticalis 60: 12: 24½, cujus Logarithmus 141766: ejus duplo 283533, adde Logarithmos crurum 581260 & 312858, fit summa 1177651: qui est Logarithmus semi-differentiæ sinuum versorum basis & differentiæ crurum: atque idem est Logarithmus sinus recti 17: 56; quem arcum, inventum secundum appellamus: est enim inventum primum quod sequitur. Differentiam crurum 13 bipartire, fient 6:30: cujus Logarithmum 2178570 duplica, & fient 4357140 pro Logarithmo dimidii sinus versi 13 gr. , & pro Logarithmo sinus recti 0 gr. 44: quem arcum 44 pro invento primo habemus. Horum inventorum aggregatum est 18 gr. 40, & ejus Logarithmus est 1139241: semi-aggregatum autem est 9 gr. 20, & ejus Logarithmus est 1819061: differentia verò est 17 gr. 12, & ejus Logarithmus est 1218382, semi-differentia verò est 8: 36, cujus Logarithmus 1900221. Adde ergo Logarithmum semi-aggregati 1819061,

Vel ad hunc Logarithmum 1218382, & fiet productum 3037443: à quo aufer Logarithmũ 1900221, & remanebunt 1137222.

Vel ad anti-logarithmum semi-differentiæ, qui est 11307, fient 1830368: hinc substrahe 693147, & restabunt 1137221.

Hos bipartire, fient 568611, cujus Logarithmi arcus est 34: 30, quem arcum duplica, & fiet basis quæsita 69 graduum.

Conversum huius problematis, ad inveniendum angulum ex datis lateribus habetur lib. Logar. Cap. 6. Sect. 8. sed partim per Logarithmos, partim per arcuum prostaphæresin.

Notandum in præcedenti & ſequentibus problematis nullâ opus eſſe caſuum obſervatione: ſpecies enim omnium partium unà cum quantitate, ex ipſo calculo prodiunt.

Sequitur alia converſio præcedentis directa.

DATAM baſin 69 gr. bipartire, fiet 34:30, cujus Logarithmus eſt 568611: quem duplica fient 1137222: cujus arcum 18 gr. 42, pro invento primo nota: ſuperioris autem Logarithmi 4357140 arcum 0 gr. 44, pro invento ſecundo nota. Horum arcuum complementa ſunt 89:16, & 71:18: horum ſemi-aggregatum eſt 80:17, & ejus Logarithmus 14449: ſemi-differentia verò 8 59, eiuſq. Logarithmus 1856956: quos adde, fient 1871405: à quibus ſubſtrahe 693147, & relinquentur 1178258, cujus arcus eſt 17 gr. 56, quem arcum, inventum tertium hîc vocamus: à cujus Logarithmo aufer Logarithmos crurum 581260 & 312858, & relinquentur 283533, quem bipartire, fient 141766 Logarithmus ſemi-anguli verticalis 60:12:24½. Totus ergo angulus verticalis quæſitus eſt 120:24:49.

Regula alia proſtaphæreticæ inventionis baſis.

SEmi-differentiam ſinuum verſorum aggregati & differentiæ crurum nota: Nota etiam ſemi-ſinum verſum anguli verticalis. Notatos hos inter ſinus rectos quære, & ſemi-differentiam ſinuum verſorum aggregati & differentiæ ſuorum arcuum in Tabula occurrentium, pro invento ſecundo ſignabis: & pro invento primo capiatur ſemi-ſinus verſus differentiæ crurum. Hæc inuenta adde, & proveniet ſemi-ſinus verſus baſis quæſitæ.

Contrà autem ex ſemi-ſinu verſo baſis, aufer primum inventum, quod eſt ſemi-ſinus verſus differentiæ crurum,

& prodibit ſecundum inventum: quod per quadratum ſinus totius ductum, & diviſum per ſemi-differentiam ſinuum verſorum aggregati & differentiæ crurum, relinquit in quotiente ſemi-ſinum verſum anguli verticalis quæſiti.

Ex quinque partibus trianguli ſphærici, quarum tres mediæ dantur, duas extremas vno opere invenire. Aut aliàs, datis duobus angulis apud baſin cum baſi, vtrumq; crus ſic habetur.

(*) ANgulorum apud baſin aggregatum, ſemi-aggregatum, differentiam, & ſemi-differentiam, unà cum ſuis Logarithmis nota. Inde Logarithmos ſemi-aggregati & differentiæ, & differentialem ſemi-baſis adde: & hinc ſubducito Logarithmum aggregati, & Logarithmum ſemi-differentiæ; & producetur differentialis, qui eſt primum inventum. Deinde Logarithmum ſemi-differentiæ, & differentialem ſemi-baſis adde: hinc aufer Logarithmum, ſemi-aggregati, & producetur differentialis, qui eſt inventum ſecundum. Inventos hos differentiales, quia veri ſunt, quære inter numeros differentiales: eorum arcus adde, & habebis crus maius; ſimiliter minorem à maiore ſubſtrahe, & habebis crus minus.

Aliter pro cruribus inveniendis.

ANgulorum apud baſin Logarithmum ſemi-aggregati, antilogarithmum ſemi-differentiæ, & differentialem ſemi-baſis adde: & aufer Logarithmum aggregati & 693147, & fiet primum inventum. Deinde Logarithmum ſemi-differentiæ, anti-logarithmum ſemi-aggregati, & differentialem ſemi-baſis adde: & hinc aufer Logarithmũ aggregati & 693147, & fiet inventum ſecundum. Cum inventis age ut ſuprà, & habebis crura.

Idem aliter.

SEcantem complementi aggregati angulorum apud baſin, duc per tangentem ſemi-baſis: productum duc primò per ſinum anguli maioris apud baſin, & fit inventum primum. Secundò duc per ſinum minoris anguli, & fit inventum ſecundum. Hos ergo inventos diviſos per quadratum ſinus totius adde, & fit tangens ſemi-aggregati crurum: ſimiliter maiorem à minore ſubſtrahe, & fiet tangens ſemi-differentiæ crurum. Eorum ergo arcuum utrumque adde, & fiet crus maius: ſimiliter minorem arcum à maiore aufer, & fiet crus minus.

Quinque partium proximarum Trianguli ſphærici datis tribus mediis, vtramque extremam vno opere, & abſq; caſuum obſervatione inquirere.

ANgulorum apud baſin, ut ſinus ſemi-differentiæ, ad ſinum ſemi-aggregati: Ita ſinus differentiæ, ad quartum quod eſt aggregatum ſinuum.

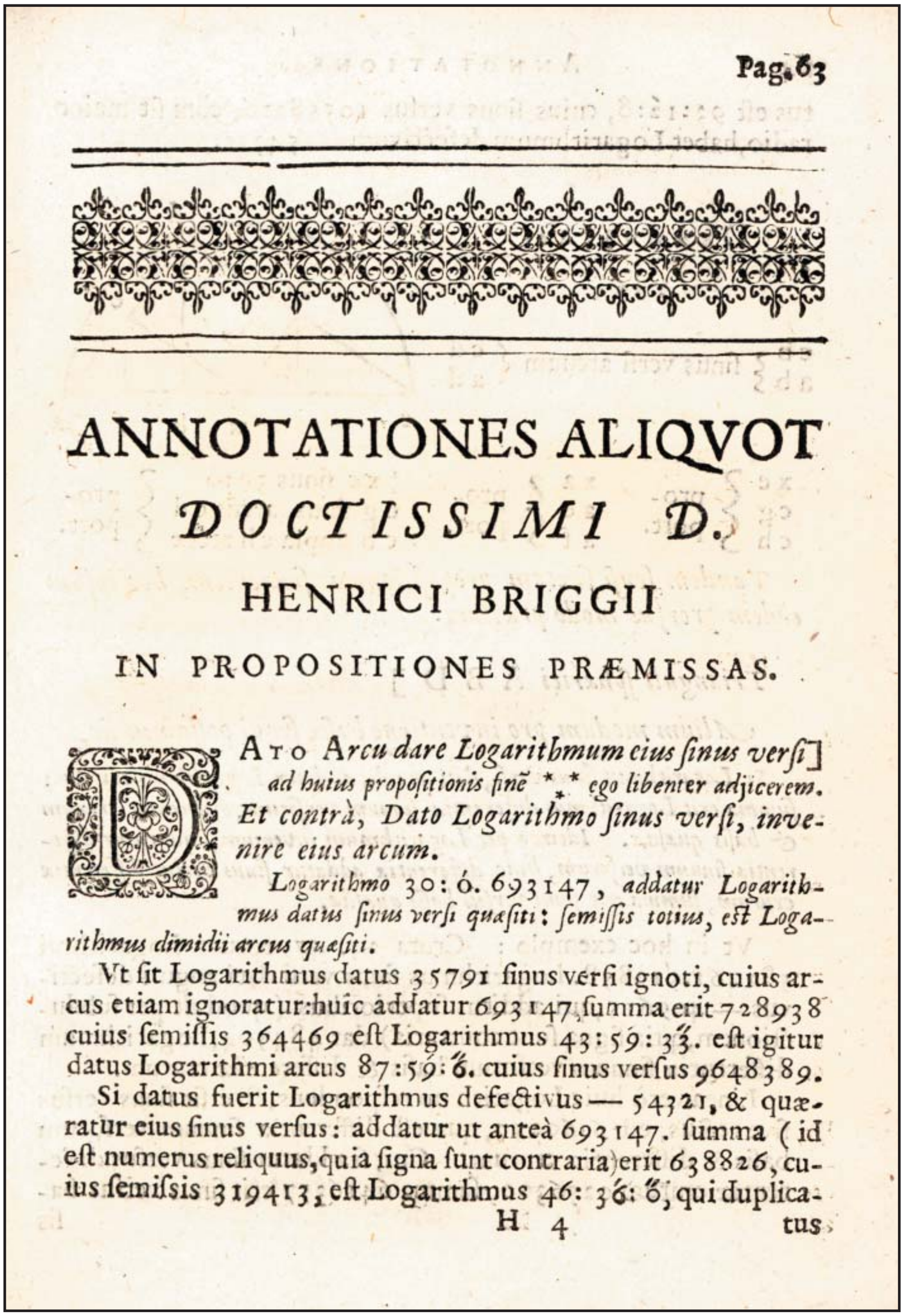
Et ut ſinus aggregati, ad hoc aggregatum ſinuum: Ita tangens ſemi-baſis, ad tangentem ſemi-aggregati crurum.

Inde ut ſinus ſemi-aggregati angulorum, ad ſinum ſemi-differentiæ: Ita tangens ſemi-baſis, ad tangentem ſemi-differentiæ crurum.

Horum inventorum tangentium arcus, è Tabula tangentium extractos adde, & prodibit crus maius: ſic minorem à maiore ſubſtrahe, & prodibit crus minus.

FINIS.

Napier, John, *Mirifici logarithmorum canonis constructi*, 1619, Edinburgh



ANNOTATIONES ALIQVOT DOCTISSIMI D. HENRICI BRIGGII

IN PROPOSITIONES PRÆMISSAS.

DATO *Arcu dare Logarithmum eius ſinus verſi*] *ad huius propoſitionis finē* * * * *ego libenter adjicerem. Et contrà, Dato Logarithmo ſinus verſi, invenire eius arcum.*

Logarithmo 30: 0. 693147, *addatur Logarithmus datus ſinus verſi quæſiti: ſemiſſis totius, eſt Logarithmus dimidii arcus quæſiti.*

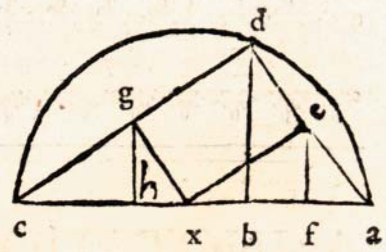
Vt ſit Logarithmus datus 35791 ſinus verſi ignoti, cuius arcus etiam ignoratur: huic addatur 693147, ſumma erit 728938 cuius ſemiſſis 364469 eſt Logarithmus 43: 59: 33. eſt igitur datus Logarithmi arcus 87: 59: 6. cuius ſinus verſus 9648389.

Si datus fuerit Logarithmus defectivus — 54321, & quæratur eius ſinus verſus: addatur ut anteà 693147. ſumma (id eſt numerus reliquus, quia ſigna ſunt contraria) erit 638826, cuius ſemiſsis 319413, eſt Logarithmus 46: 36: 6, qui duplicatus

In this section Henry Briggs adds his remarks to the previous discussion of spherical triangles and related problems.

tus est 93:12:6, cuius ſinus verſus 10558216, cùm ſit maior radio, habet Logarithmum defectivum — 54321.

DEMONSTRATIO.



cb, ab } ſinus verſi arcuum { cd, ad

xc, cg, ch } proport.   xa, ae, af } proport.   ½ xc ſinus 30:6, c g ſinus ½ arcus c d, c b dupla c h rectæ } proport.

Tandem ſenſi ſextam propoſitionem ſequentem, hoc ipſum eodem prorſus modo præſtare.

Trianguli ſphærici A B D]

Alium modum pro inventione baſis ſequi poſſumus ſic.

Si Logarithmus ſinus verſi, dati anguli, addatur Logarithmis crurum: ſumma erit Logarithmus differentiæ ſinuum verſorum, differentiæ crurum & baſis quæſitæ. Idcircò per Logarithmum inventum, quæratur differentia ſinuum verſorum, huic differentiæ addatur ſinus verſus differentiæ crurum, ſumma erit ſinus verſus baſis quæſitæ.

Vt in hoc exemplo: Crura 34. 47; eorum Logarithmi 581261.312858. Logarithmus ſinus verſi dati anguli defectivus — 409615, qui additus ſuperioribus (quod fit per ſubductionem, quia ſigna ſunt contraria) dat 484504, Logarithmum differentiæ ſinuum verſorum baſis & differentiæ crurum.

Linea verò huic Logarithmo reſpondens, ſive ſit ſinus verſus ſive rectus, eſt 6160057, quæ eſt differentia ſinuum verſorum baſis & differentiæ crurum. Cui, ſi addatur ſinus verſus differentiæ crurum 0256300, ſumma 6416357 erit ſinus verſus baſis

ſis quæſitæ: qui ablatus è radio, relinquit 3583643 ſinum rectum complementi baſis 21: 6. eſt igitur baſis 69: 6.

Et contrà datis tribus lateribus, invenitur angulus quilibet.

Si è Logarithmo differentiæ ſinuum verſorum, baſis & differentiæ crurum; auferantur Logarithmi crurum, reliquus erit Logarithmus ſinus verſi anguli quæſiti.

Vt in priori exemplo è Logarithmo 484504 auferatur 894119 reliquus erit Logarithmus defectivus — 409615 qui dabit nobis ſinum verſum anguli quæſiti. 120: 24: 49.

Ex quinque partibus trianguli ſphærici] *Hæc propoſitio omnino eadem eſſe videtur cum vltima, quæ ad finem adjecta, eodem modo à me notatur ſic* (*) *Hanc ego præſtantiſſimam eſſe lubentiſſimè exiſtimo. Sunt autem tres operationes, quæ in vltima magis ſunt diſtinctæ, earum duas priores in vnam conjicio, ſic.*

Sunto data baſis 69. 6.

Anguli ad baſin { 42: 29: 59 / 31: 6: 5

73: 36: 4	ſumma
36: 48: 2	ſemi-ſumma
53: 11: 58	comp. ſemiſummæ
11: 23: 54	differentia
5: 41: 57	ſemi-differentia
84: 18: 3	comp. differentiæ

			Logarithmi
1. Prop.	Sinus ſemi differentiæ	5: 41: 57	23095560
	Sinus ſemiaggregati	36: 48: 2	5124410
	Sinus differentiæ	11: 23: 54	16213641
	Summa ſinuum		— 1757509
2. prop.	Sinus aggregati	73: 36: 4	415312
	Summa ſinuum		— 1757509
	Tangens ſemibaſis	34: 30: 0	3750122
	Tang. ſummæ crurũ	40: 30	1577301

			Logarithmi
3. prop.	Sinus aggregati angulorum	36: 48: 2	5124410
	Sinus semidifferentiæ angulorũ	5: 41: 57	23095560
	Tangens semibasis	34: 30: 0	3750122
	Tangens differentiæ crurum	6: 30: 0	21721272

40: 30
6: 30

47: 0 } crura.
34: 0 }

Hæ sunt operationes ab autore traditæ. Ego verò, vnam pro duabus primis constituo, tertiam verò seruo.

			Logarithmi
Proport.	Sinus compl. summæ angulorũ	53: 11: 58	2222368
	Sin. comp. differẽtiæ augulor.	84: 18: 3	49553
	Tangens semibasis	34: 30: 0	3750122
	Tangens semisummæ crurum	40: 30: 0	1577307

ALIVD EXEMPLVM.

Sunto datus angulus 47 : 0.

Crura comprehendentia { 59: 35: 11 / 31: 6: 5

90: 41: 16 summa
45: 20. 38 semisumma
44: 39: 22 compl. semisummæ

28: 29: 6 differentia
14: 14: 33 semidifferentia
75: 45: 27 com. semidifferentiæ

			Logar.
1. prop.	Sinus compl. summæ crurum	44: 39: 22	3526118
	Sinus compl. differentiæ crurũ	75: 45: 27	312192
	Tangens compl. anguli vertic.	66: 30	— 8328403
	Tang. sum. angul. ad basim	72: 30	— 11452329

			Logar.
2.prop.	Sinus ſemiſummæ laterum	45:26:38	3406418
	Sinus ſemidifferentiæ laterum	14:14:33	14023154
	Tangens com. ſemiang. vert.	66:30: 0	— 8328403
	Tang. ſemidiff. ang. ad baſim	38:30	2288333

72 : 30
38 : 30
____
111 : 0 } Anguli ad baſim
34 : 0 }

Atque hæc omnia conſtantiſſimè ſervantur, ſive dati fuerint duo anguli, cum latere interiecto: ſive duo crura, cum angulo comprehenſo. Hoc tantum intereſt, quod tertium proportionis locum, in utraque operatione: illic, Tangens ſemibaſis occupat: hic, Tangens compl. ſemiſsis anguli verticalis. In his exemplis, ſi Tangens vel ſumma ſinuum, ſit maior Radio circulari: Logarithmus eſt defectivus, & habet virgulam præcedentem ſic — 8328403.

Idem aliter]

Hos ergo inventos diviſos per quadratum ſinus totius adde) *Ego ſic potius ſcriberem, quò res eſſet manifeſtior.* Horum ergo inventorum, per quadratum ſinus totius diviſorum, quotos adde, & fiet Tangens, &c.

Hæc propoſitio veriſsima eſt, vt & proxime antecedens; ſed illa per Logarithmos commodiſsimè expedietur, hæc tota, vix poterit Logarithmorum operationes admittere: quia quoti ſunt addendi & auferendi, vt Tangentes inveniantur. Logarithmorum autem vſus cernitur in proportionalibus, & idcircò in multiplicatione & diviſione: non autem in additione aut ſubductione.

FINIS.