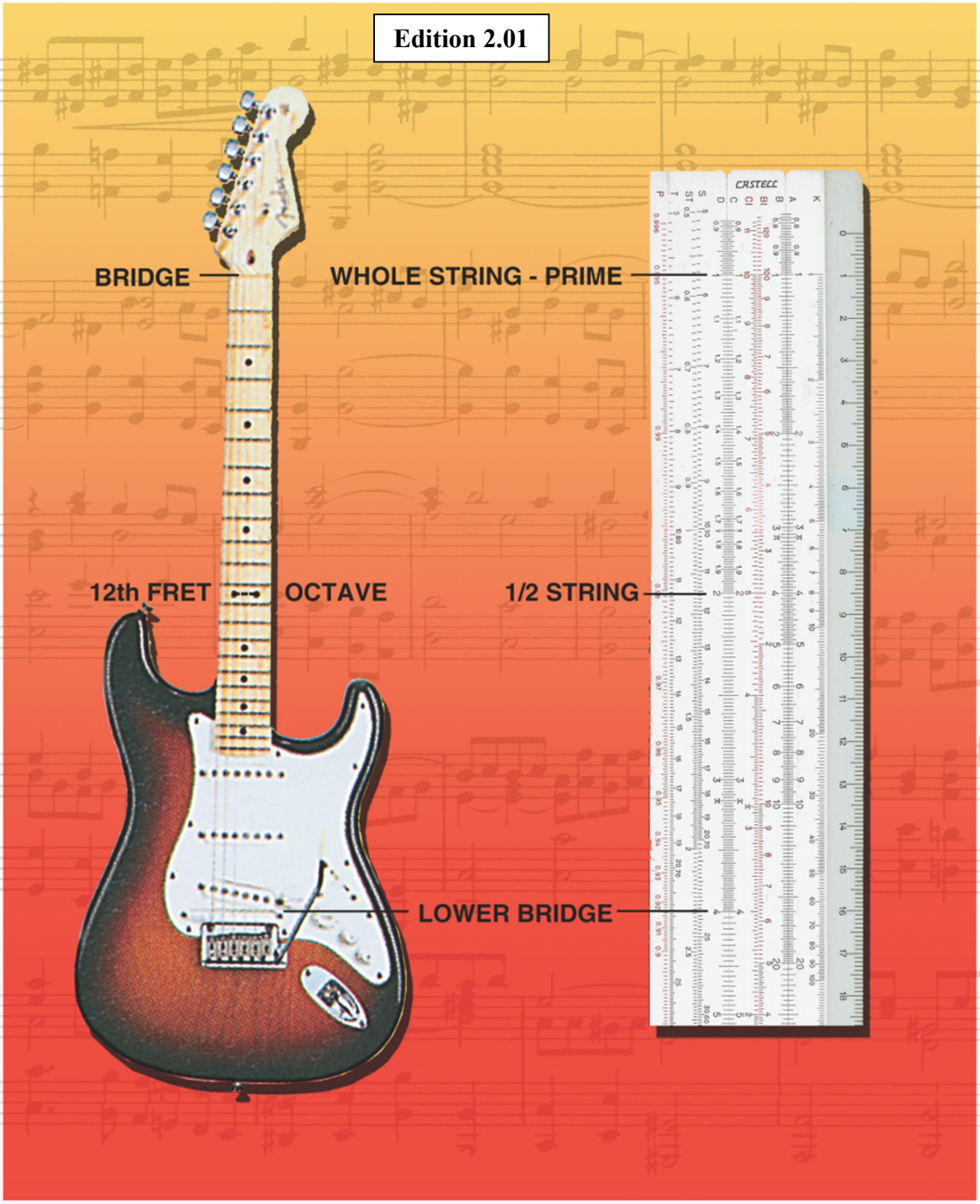
# Calculating with Tones:

## The Logarithmic Logic of Music

Edition 2.01




By Klaus Kuehn and Rodger Shepherd

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# The Logarithmic Logic of Music

by Klaus Kuehn
and Rodger Shepherd († 2020 – in memoriam)

**Edition 2.01**
**Spring 2023**

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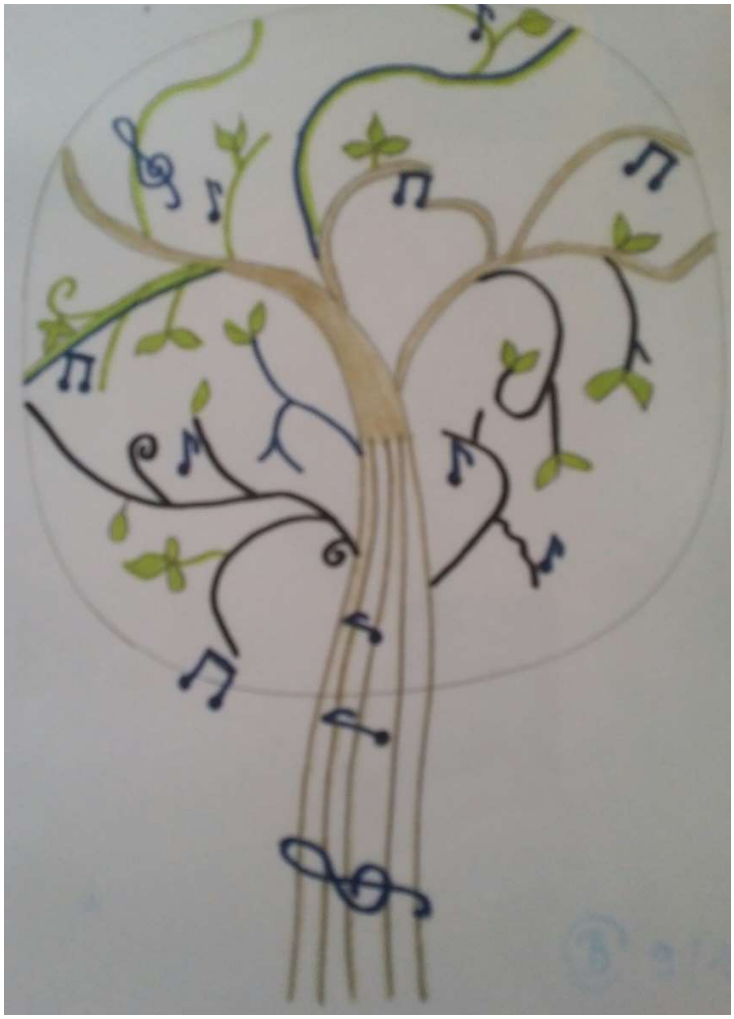

Bettina Compes: Der Notenbaum – Tree of Notes

Many thanks to Bettina!

This 2nd edition of “Calculating with Tones” is dedicated to my son Jasper Leonard and to his wife Nicole as well as to their offspring Remo.

## Preface for this Educational edition

What is New?

Since the first edition was published in 2009 more new literature as well as new online media has appeared about “Calculating with Tones”. So for this second edition a careful selection of the existing material was necessary in order to avoid redundancy but equally new and harder to come by aspects from the last 13 years were catalysts for updating the first edition.
The content is structured the same way but it is now complemented with the background to the history of Music Theory which drove the development of calculating methods for optimising the subdivision of a musical scale. On the other hand some rare “newly discovered” educational specialities and devices supporting teachers and musicians when working with EQUAL TONE settings are now also incorporated.

Despite the rich pool of content available via the Internet this manuscript can obviously only cover a small part of the existing and available sources and material. So the focus was on including the core information.

In the first 3 chapters the reader will be introduced to the basics of tones and the intervals (distances) between those tones.
New in this 2nd edition is the history of the theory behind musical developments. This will be covered in some “rare” details in chapter 4.
Chapter 5 and 6 are devoted to the role of logarithms in music. There will be an introduction to the history of tone logarithms as well as to some newly discovered devices like the “Musical Graphics” from Michael Stifel and Abdias Trew [Troy]. Additionally the reader will find details about a mesolabium, some newly “discovered” musical sectors, slide discs, slide charts, and computer programs for calculating with tones. The last part of chapter 6 shows the usage of numbers by composers.This edition ends with a summary, acknowledgments and a bibliography.

The full 2nd Edition (146 pages) of ‘Calculating with Tones - The Logarithmic Logic of Music’ will be solely distributed as a private and educational “print only” Creative Commons publication. Requests for a list of additional readings, supplements, relevant LINKS and material which might be protected by copyright will only be available per keyword. Such requests should be made to the author via www.Collectanea.eu.

My thanks especially go to Michael Casey but also to all the contributing members of the very active InternationalMusicologicalSociety (IMS) Study Group Musical Diagrams: [Musical Diagrams 2022] and to the Zurich University of the Art offering a great deal of in-depth and very specific information on calculating with tones at https://sound-colour-space.zhdk.ch/ .

My sincere thanks also go to the Oughtred Society and its president Robert de Cesaris who triggered the compilation of this second edition.

Very much appreciated and honoured is the enormous and always valuable co-editor/co-author like support provided by David Rance.

Klaus Kühn, Kompilator –
collecting, selecting (the toughest part) and presenting available knowledge and sources in a meaningful, clear, historical, attractive/interesting and educational way.

## Preface to the first edition 2009

The topic of "Calculating with Tones" requires perspectives from several different disciplines, such as physics, mathematics, music theory, and psychophysics. With this in mind, we have provided some brief, but we hope sufficiently detailed, background information from these various disciplines. This allows an understandable discourse on the connections between calculation *of* tones and calculation *with* tones.

The interest in this topic was stimulated first by a "Slide Rule for Organ Pipes (System Rensch)" which was offered at an auction (and which KK failed to acquire) and later by encouragement from a fellow slide rule collector. As a result we looked more closely at the relationship between mathematics and musical tones. The literature on this topic is essentially complete, and there is not much more to be said or written. However, this literature is not easily accessible for most people. Therefore in the monograph that follows we would like to offer to the reader all that we have learned in a condensed and clearly arranged form. All the relevant references are cited in the bibliography and are at hand. Of course the results of searches on the Internet have also provided material for this short review. At first the results from our Internet searches were meagre. However, as we gained familiarity with the subject, the yield became voluminous (over 4500 citations) so that we had to avoid losing sight of the forest for the trees.

It has been repeatedly said that musical people also have a good understanding of mathematics. It is not the purpose of this paper to investigate this combination or association. This discourse should merely permit the reader to recall some relationships (that perhaps have faded from memory) or become aware of them for the first time. Of course the topic might not have interested us as much if logarithms had not played such an essential role in music.

This monograph is the updated and significantly expanded version of the first edition from 2009.
It now addresses the following questions:

1. How is a musical scale constructed? How were they conceived?
2. What contribution did Pythagoras make 2500 years ago?
3. How has the monochord helped?
4. What is the difference between pure and tempered tuning?
5. What is the significance of the Pythagorean comma?
6. Are there mathematical tasks/problems in music?
7. What is meant by the circle of fifths?
8. How do dyadic logarithms relate to music?
9. What advantage is offered by measuring in "Cent"?
10. What does the Weber-Fechner law assert?
11. What are the implications of tone-logarithms?
12. Why are the keys of a keyboard comparable to a slide rule?
13. How and why do the intervals between the frets on a guitar narrow with higher pitch?
14. How do preferred numbers relate to oscillation ratios of tones?
15. What does Music History tell us?
16. What did Music Theorists/Historians intent?
17. Why were musical SR's so helpful?
18. What makes the difference between consonance and dissonance?
19. What is so special with Western Music?
20. What does Ethnomusicology look for?

## Table of Contents


***Preface for this Educational edition – what is new?***
***Preface to the first edition 2009***

1. ***Introduction*** ..... ***8 - 10***

***2. Pythagoras*** ..... ***11***
2.1. Monochord ..... ***11***
2.1.1. Robert Fludd (1617 ..... ***14***
2.2. The Origin of the Pythagorean tone scale ..... ***15***
2.2.1. Tromba Marina (one string) ..... ***19***
2.2.2. Guzheng – Chinese zither (multi string) ..... ***21***
2.2.3. Juan Caramuel (multi string) ..... ***22***
2.3. Geometrical calculation of the Pythagorean tone scale ..... ***23***
2.4. Geometrical calculation of a square root ..... ***24***

***3. Intervals of Pitch*** ..... ***25***
3.1. Diatonic or pure? ..... ***25***
3.2. The Circle of Fifths ..... ***29***
3.2.1. Circulo Musico - Hartung (1749) ..... ***32***
3.3. Tempered Pitch ..... ***34***
3.4. Musical scales of other cultures ..... ***39***
3.4.1. Greece ..... ***39***
3.4.2. India ..... ***41***

***4. Tools and Graphics used in Music Theory before Logarithms*** ..... ***42***
4.1. From Pythagoras (500 BC) to Boethius (500 AD) ..... ***43***
4.2. Guido's Hand, tone notation and Solmization (~ 1000) ..... ***45***
4.3. Berno von Reichenau (~ 1000) ..... ***47***
4.4. Johannes de Muris (1355) and Glareanus/ Henricus Loriti (1547) - Greek Scale names ..... ***48***
4.5. Geometrical-mechanical tool (Mesolabio) described by Gioseffo Zarlino 1558, 1571 ..... ***49***
4.6. Ambrosius Wilfflingseder (1563) ..... ***50***
4.7. Maternus Beringer (1610) ..... ***51***
4.8. Abdias Trew (1597 – 1669) ..... ***52***
4.9. Sigmund Theophil Staden´s Disc ~ 1650 ..... ***54***

***5. The Role of Logarithms*** ..... ***55***

5.1. Michael Stifel – Arithmetica Integra 1544 ..... ***55***

5.2. Dyadic logarithms and "Cent" ..... ***58***

5.3. Tone logarithms ..... ***69***

5.3.1. History of Tone Logarithms – Time Tables and Applications ..... ***69***

5.3.2. Tables and applications of musical logarithms (a selection) ..... ***72***

5.3.2.1. Dutch ..... ***72***

5.3.2.2. Italian and Spanish ..... ***73***

5.3.2.3. French ..... ***75***

5.3.2.4. English ..... ***78***

5.3.2.4.1. Alexander Ellis (1814 – 1890) ..... ***78***

5.3.2.4.2. Working with CENT: A Survey, Fred Lieberman ..... ***80***

5.3.2.5. German ..... ***81***

5.3.2.5.1. Cent table ..... ***82***

5.3.2.5.2. Frequency and Cent relationship ..... ***83***

5.3.2.5.3. Helmut K.H. Lange ..... ***85***

5.4. The spacing of frets on a guitar..... ***87***

5.5. Tones and Preferred numbers ..... ***90***

5.6. The Weber-Fechner Law or the Quantification of Sensation (Basilar Membrane) ..... ***92***

***6. Calculating Devices and Slide Rules for music(ians)*** ..... ***94***

6.1. Geometrical-Mechanical Approaches ..... ***100***

*6.1.1.* Mesolabio by Gioseffo Zarlino 1571 ..... ***100***

6.2. Musical Graphics by Abdias Trew – 1635 ..... ***103***

6.3. Circular Pitch Diagrams like the Musical Compass English 1684 ..... ***108***

6.4. Sectors Musicalis ..... ***109***

6.4.1. Sector Musicalis – before 1650 ..... ***109***

6.4.2. Compas Harmonico – Joseph Zaragoza 1675 ..... ***111***

6.4.3. Sectors Musicalis – around 1750 ..... ***114***

6.4.4. Diagonal Scale for monochord scaling – 1808 ..... ***117***

6.4.5. Slide Discs for transposing musical notes ..... ***118***

6.4.6. Slide Charts ..... ***119***

6.4.7. Computer Programs ..... ***122***

6.5. Composers and Musicians playing/working with numbers ..... ***123***

6.5.1. Johann Sebastian Bach (1685 – 1750) – "Bach en het getal" Bach and numbers ..... ***123***

*6.5.2.* Wolfgang Amadeus Mozart (1756 – 1791) "Mozart's Würfelmusik - Musical Dice Games" ***125***

6.5.3. Arnold Schoenberg – Twelve-tone devices ..... ***128***

6.5.4. Harry Partch (1901 – 1974) – "Chromelodeon" ..... ***131***

*6.5.5.* Other examples: calculating and producing bells ..... ***131***

6.6. The other way around: program for transferring music to numbers/coloured computer graphs.. ***132***

***7. Summary*** ..... ***134***

***8. Acknowledgements*** ..... ***138***

***9. Bibliography*** ..... ***139 -145***